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# EXTRACTION OF VEHICLE TRANSFER FUNCTIONS FROM NOISY FLIGHT TEST DATA VIA A DISCRETE DECOUPLED TECHNIQUE

PHASE II



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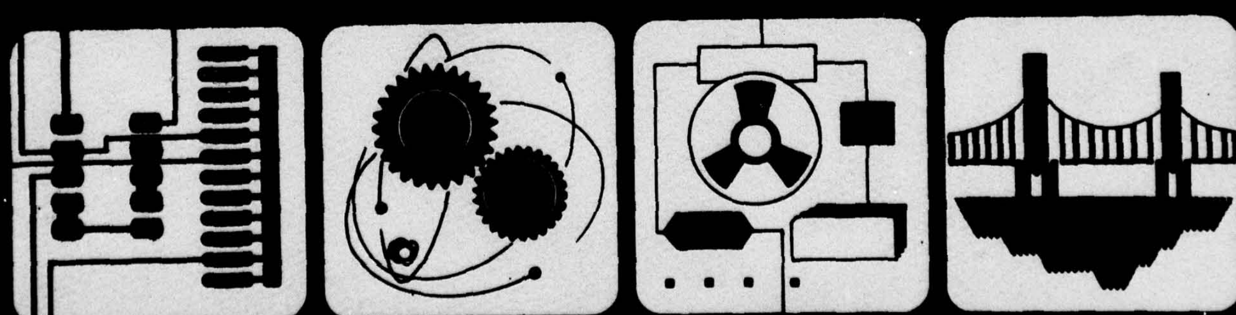

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A method is developed for estimating the parameters of linear transfer functions describing a multi-input multi-output dynamic system from noisy flight test data. The method is demonstrated on real and synthetically generated underwater vehicle data.

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# EXTRACTION OF VEHICLE TRANSFER FUNCTIONS FROM NOISY FLIGHT TEST DATA VIA A DISCRETE DECOUPLED TECHNIQUE

(Phase II)

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## INDEX

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# I. INTRODUCTION

Under previous contracts (N61331-74-C-0027 and N61331-74-C-0070) and the present contract (N61331-75-C-0012) a new technique for the identification of vehicle transfer functions from flight test data has been developed and tested. The importance of dynamics identification arises because the resulting model provides i) a quantitative measure of the stability of the vehicle, ii) a means to predict the vehicle response to new maneuvers, and iii) a basis for designing and evaluating a control system for the vehicle, through simulation or analytical procedures. To elaborate on the latter consider a towed sonar vehicle for which the deviations of the roll, pitch and yaw angles -- i.e. the vehicle attitude angles -- are required to be controlled carefully. Clearly, the dynamics of the vehicle must first be identified before the analytical design of an optimum controller, or the subsequent evaluation of the overall system through simulation, can be carried out.

Salient features of the new identification technique GRAM are summarized below:

(a) Noniterative - The method is non-iterative [1], and therefore avoids the problems of convergence often associated with iterative techniques.

(b) Noiseworthy - The method is noiseworthy [2]. Accurate estimation can therefore be achieved even in the presence of high levels of noise [2] provided that the system response contains all of its modes in a reasonably observable manner. This success can be attributed to the fact that the method utilizes signals generated via measurement filters whose poles can be chosen appropriately by the test engineer.

(c) Multiinput-Multioutput - The method is applicable to simultaneous analysis of different input and output variables of the vehicle. This results in a common denominator model for all of the output variables involved, a very desirable feature both from the theoretical as well as the practical viewpoints. A state variable formulation of the system the vehicle is also facilitated, indeed made possible, because of this fact. Another advantage accruing from this feature is that an accurate detection of all modes becomes possible provided that each mode manifests itself reasonably, or strongly, in at least one of the output variables.

(d) State Model - As stated above, a state model can be obtained for the vehicle even if only a subset of state variables is measured; this includes the possibility of using only one output variable.

(e) Non-Zero Initial Conditions - The effect of non-zero initial conditions can be taken into account and their contribution separated from the input-output dynamics.

(f) Modeling under mild nonlinearity - When the test manuever is such that the effect of nonlinearities is mild, that is the linear dynamics is dominant it isfound that the GRAM technique is able to detect the linear part of the model with sufficient accuracy. Of course, in a practical test it may not be known a priori that the response is dominated by linear dynamics; therefore the usefulness of the remark must be interpreted cautiously.

(g) Unstable Vehicle Dynamics - The method is applicable to the identification of unstable linear systems. More precisely, consider that the unstable system is embedded in a feedback loop, manual or

automatic, such that the overall system behaves in a stable manner. Then it is found that the application of GRAM technique to the input-output data of the unstable vehicle yields its dynamics, including the poles in the right-half s-plane, with considerable accuracy.

Under the present contract, #N61331-75-C-0012, the following segments of research work were performed.

1. Consideration of non-zero initial conditions and state model formulation.

2. Application of GRAM technique to flight test data of the DSRV (deep submergence research vehicle). A special feature of this work was the detection of a weak oscillatory mode from the short duration test data that was available.

3. Adaptation and testing of GRAM technique to unstable systems.

4. Cascading of the GRAM technique with the NASA (Taylor) Systems Identification Technique [3].

5. Application of GRAM to flight test data generated from the Navy standard program 'DYNAMIC' in the presence of mild nonlinearities.

The above research efforts, which encompass theoretical developments, computer implementation, and testing, are described in the succeeding chapters.

# II. MULTIVARIABLE IDENTIFICATION OF VEHICLE DYNAMICS WITH NON-ZERO INITIAL CONDITIONS, STATE-MODEL

The purpose of this chapter is to discuss the identification of vehicle dynamics under non-zero initial conditions, estimating these unknown initial conditions, and to generate a state-variable model from the identified transfer function matrix. A computer program written in FORTRAN IV was developed and tested. A listing of the complete program, MGRAM, and a card deck for the same are enclosed. After a theoretical discussion of the method the results of identification performed on the simulated flight test data of a six-man submersible are presented.

## 2.1 COMMON MODE FORMULATION

Consider that the vehicle dynamics is described by the state equations

$$x(k+1) = F^1 x(k) + F^2 u^1(k) \tag{1a}$$

$$v(k) = F^3 x(k) + F^4 u^1(k) \tag{1b}$$

where $x(k)$ is the N-dimensional vector of state variables, $v(k)$ is the m-dimensional vector of outputs, and $u^1(k)$ is an r-dimensional vector of known inputs. The constant matrices $F^1$, $F^2$, $F^3$ and $F^4$ have appropriate dimensions. In general, the measured outputs are corrupted by bias and noise:

$$y(k) = v(k) + f + e(k) \tag{1c}$$

The z-domain description of this syste- is given by

$$x(z) = (zI-F^1)^{-1}[F^2 u^1(z) + x(0)z] \tag{2a}$$

$$y(z) = F^3 x(z) + F^4 u^1(z) + f\ (1-z^{-1})^{-1} + e(z) \tag{2b}$$

or, upon combining these equations,

$$y(z) = F^3(zI-F^1)^{-1}[F^2 u^1(z)+x(0)z] + F^4 u^1(z) + f\ (1-z^{-1})^{-1} + e(z) \tag{3}$$

where I denotes the identity matrix of order N. Upon rearrangement (3) becomes

$$y(z) = [H^1(z) \;\vdots\; h(z) \;\vdots\; f] \begin{bmatrix} u^1(z) \\ 1 \\ (1-z^{-1})^{-1} \end{bmatrix} + e(z) \tag{4}$$

In (4),

$$H^1(z) = F^3(zI-F^1)^{-1}F^2 + F^4 \tag{5}$$

$$h(z) = zF^3(zI-F^1)^{-1}x(0) \tag{6}$$

The important observation arises that the case of non-zero initial conditions has been cast into a form that is analogous to the case of zero initial conditions and zero bias. We now define

$$H(z) = [H^1(z) \mid h(z) \mid f] \tag{7}$$

$$u(z) = [u^1(z)^T \mid 1 \mid (1-z)^{-1}]^T \tag{8}$$

where the superscript T denotes the transpose operation. Of course, if the initial conditions and the bias are zero then the formulation becomes

$$H(z) = H^1(z) \tag{9}$$

$$u(z) = u^1(z) \tag{10}$$

Regardless of the above details we can now state the problem as follows.

Problem statement - Consider the model (see Fig. 1 )

$$y(z) = H(z)\, u(z) + e(z) \tag{11}$$

where H(z) is an $m \times r$ matrix with rational entries having a common denominator; i.e. its entries have the form

$$h_{ij}(z) = B_{ij}(z) / A(z) \tag{12}$$

$$= \frac{b_{ij0} + b_{ij1}z^{-1} + \ldots\ldots + b_{ijN_j}z^{-N_j}}{1 + a_1z^{-1} + \ldots\ldots + a_Nz^{-N}} \tag{13}$$

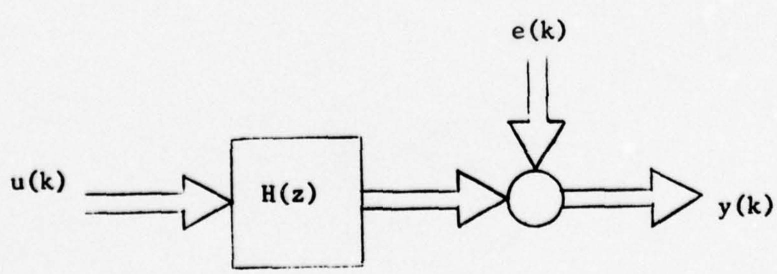



Fig. 1. Vehicle model

Given the model (11), the sequences $y(k)$ and $u(k)$ over a set of points $0,1, \ldots, L$ determine the numerator parameters $b_{ijn}$, $i=1, \ldots, m$, $j=1, \ldots, r$, $n=1, \ldots, N$ and the denominator parameters $a_n$, $n=1, \ldots, N$ such that the estimated model provides a best fit in some sense.

The error vector $e(k)$ may arise from measurement errors, insufficiently high order of approximation, or simply incorrect modeling of the process. Suitable assumptions must therefore be made on the error vector $e(k)$ depending on the specific application.

## 2.2 DEVELOPMENT OF A KEY EQUATION

Before dealing with the problem stated above let us point out that this formulation emphasizes common modes between the various outputs. This is in contrast to the formulation in [4] where each output was considered to have its own denominator dynamics. That freedom leads to undesirable results when it is known a priori that the outputs (i.e., the different entries of the output vector $y(k)$) share common modes. Such is the case, for example, when $y(k) = [u(k), \theta(k), w(k)]$, the longitudinal variables of a submersible.

Indeed, we have assumed $H(z)$ in (11) to have the common-mode form

$$H(z) = [B_{ij}(z)/A(z)]_{mxr}$$

We now proceed with the development by first multiplying equation (11) by $A(z)$. This yields

$$A(z)y(z) = B(z)u(z) + A(z)e(z) \tag{14}$$

It is instructive to look at equation (14) explicitly for the two-input, two-output case. For, then,

$$\begin{bmatrix} A(z)y_1(z) \\ A(z)y_2(z) \end{bmatrix} = \begin{bmatrix} B_{11}(z) & B_{12}(z) \\ B_{21}(z) & B_{22}(z) \end{bmatrix} \begin{bmatrix} u_1(z) \\ u_2(z) \end{bmatrix} + \begin{bmatrix} A(z)e_1(z) \\ A(z)e_2(z) \end{bmatrix} \tag{15}$$

which, in the time-domain can be written as

$$\begin{bmatrix} y_1(k) & \dots & y_1(k-N) \\ y_2(k) & \dots & y_2(k-N) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \cdot \\ \cdot \\ a_N \end{bmatrix} =$$

$$\begin{bmatrix} u_1(k)\dots u_1(k-N_1) & u_2(k)\dots u_2(k-N_2) & 0 \;..\; 0 & 0 \;..\; 0 \\ 0 \;\;\dots\; 0 & 0 \;\;\dots\; 0 & u_1(k) \;..\; u_1(k-N_1) & u_2(k)\,..\,u_2(k-N_2) \end{bmatrix} \begin{bmatrix} b_{110} \\ \cdot \\ \cdot \\ b_{11N_1} \\ b_{120} \\ \cdot \\ \cdot \\ b_{12N_2} \\ b_{210} \\ \cdot \\ \cdot \\ b_{21N_1} \\ b_{220} \\ \cdot \\ \cdot \\ b_{22N_2} \end{bmatrix}$$

$$+ \begin{bmatrix} e_1(k) & \dots & e_1(k-N) \\ e_2(k) & \dots & e_2(k-N) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \cdot \\ \cdot \\ a_N \end{bmatrix} \tag{16}$$

For the general multi-input, multi-output relation in (14) we define

$$\nu = \sum_{j=1}^{r} (N_j+1)$$

| | |
|---|---|
| $A = [1 \; a_1 \; \ldots \; a_N]^T$ | N+1 column vector |
| $B^{(ij)} = [b_{ij0} \; \ldots \; b_{ijN_j}]^T$ | $N_j+1$ column vector |
| $B^{(i)} = [B^{(i1)T} \; B^{(i2)T} \; \ldots \; B^{(ir)T}]^T$ | $\nu$ column vector |
| $B = [B^{(1)T} \; B^{(2)T} \; \ldots \; B^{(m)T}]^T$ | $m\nu$ column vector |
| $y^{(i)}(k) = [y_i(k) \; y_i(k-1) \; \ldots \; y_i(k-N)]$ | N+1 row vector |
| $e^{(i)}(k) = [e_i(k) \; e_i(k-1) \; .. \; e_i(k-N)]$ | N+1 row vector |
| $u^{(j)}(k) = [u_j(k) \; u_j(k-1) \; .. \; u_j(k-N_j)]$ | $N_j+1$ row vector |
| $U(k) = [u^{(1)}(k) \; u^{(2)}(k) \; \ldots \; u^{(r)}(k)]$ | $\nu$ row vector |

Using the above notation we can write (14) in the time domain as m equations of the form

$$y^{(i)}(k)A = U(k)B^{(i)} + e^{(i)}(k)A \tag{17}$$

where i=1, 2 ... m, or as one equation as follows:

$$\left[\begin{array}{c|cccc} y^{(1)}(k) & -U(k) & 0 & \ldots & 0 \\ y^{(2)}(k) & 0 & -U(k) & \ldots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ y^{(m)}(k) & 0 & 0 & \ldots & -U(k) \end{array}\right] \left[\begin{array}{c} A \\ \hline \\ B \\ \\ \end{array}\right] = \left[\begin{array}{c|c} e^{(1)}(k) & \\ e^{(2)}(k) & \\ \cdot & 0 \\ \cdot & \\ \cdot & \\ e^{(m)}(k) & \end{array}\right] \left[\begin{array}{c} A \\ \hline \\ B \\ \\ \end{array}\right] \tag{18}$$

Denote the m x $(N+1+m\nu)$ dimensional matrices on the left and right hand sides of equation (18) as S and E, respectively, and the $(N+1+m\nu)$ dimensional parameter vector $[A^T \mid B^T]^T$ as $\gamma$. Then, we can rewrite (18) as

$$S(k)\gamma = E(k)\gamma \tag{19}$$

Note that only the first $(N+1)$ columns of $E$ can have nonzero entries; the remaining columns are zero. In fact, we can express $E$ as

$$E(k) = [\, E_1(k) \,|\, 0 \,] \tag{20}$$

where $0$ is an $m \times \nu$ dimensional matrix of zeros.

Equation (19) is the key equation we wished to obtain in this section. The development of the method, presented in the succeeding sections, is based upon this relation.

## 2.3 SOLUTION

Difficulty in solving (19) for the parameter vector $\gamma$ arises because the error vector $e(k)$, and therefore the matrix $E(k)$, is generally unknown. Suitable assumptions on $e(k)$ must therefore be made. We shall solve (19) for increasingly less restrictive assumptions on $e(k)$.

### 2.3.1 Ideal Data ($e(k) \cong 0$)

When the error in observations $y(k)$ can be assumed to be zero, equation (19) becomes

$$S(k)\,\gamma = 0, \qquad k=0,1,\,..,\,K \tag{21}$$

The innerproduct of (21) when summed over all k yields

$$\gamma^T G\, \gamma = 0 \tag{22}$$

where

$$G = \sum_{k=0}^{K} S^T(k)\, S(k) \tag{23}$$

is the Gram matrix of measurement signals. Note that $G$ is real symmetric, hence all of its eigenvalues are real and nonnegative. Equation (22) indicates that one of the eigenvalues is zero; call the corresponding eigenvector as $\gamma^{(0)}$. Then the desired parameter vector is given by

$$\gamma = \frac{\gamma^{(0)}}{\gamma^{(0)}(1)} \tag{24}$$

2.3.2 Additive White Noise in Observed Signals

Suppose the error in observations can be assumed to be zero mean white noise, i.e.,

$$\mathcal{E}\,[e(k)] = 0$$

$$\mathcal{E}\,[e(k)\ e(\ell)] = D\ \delta_{k\ell}$$

where $\mathcal{E}$ denotes the expectation operator, D a diagonal matrix, and $\delta$ the usual Knoecker delta. Let the diagonal entries of D be denoted as $d_{ii}$ and the trace as d. Then the inner product of (19) with itself when summer over k=0, ..., K yields

$$\gamma^T G\, \gamma = \gamma^T P\, \gamma \tag{25}$$

Here,

$$G = \sum_{k=0}^{K} S^T(k)\ S(k), \tag{26}$$

$$P = \sum_{k=0}^{K} E^T(k)\ E(k) \tag{27}$$

Note that

$$\mathcal{E}\,P = \sum_{k=0}^{K} \sum_{i=1}^{m} R^{(i)} \tag{28}$$

$$= \beta C = \beta \begin{bmatrix} C^{(11)} & 0 \\ 0 & 0 \end{bmatrix} \tag{29}$$

where $R^{(i)}$ is the correlation matrix of the ith row of the matrix E(k), $\beta$ = (K+1)d and $C^{(11)}$ is an identity matrix of order N+1. We shall solve the problem of estimating $\gamma$ as that of solving

$$\hat{\gamma}^T\ G\hat{\gamma} = \beta\hat{\gamma}^T P\ C\hat{\gamma} \tag{30}$$

such that $\beta$ is minimized. (Note that for $\beta$ = 0 the problem reduces to case A.) That the estimate so obtained makes sense is justifiable in two ways.

- When the number of observations is large, P is expected to tend to $\beta C$; thus, in the absence of the knowledge of P we may replace it by $\beta C$ in equation (25) to obtain (30).
- If, instead of using successive values of k in the summations

(26) and (27), we use k=0, N+1,2(N+1), ... then it can be shown that (30) provides a maximum likelihood estimate [16] for the parameter vector under the assumption of gaussian white noise.

Since a non-zero solution is required, solving (30) is clearly equivalent to solving the generalized eigenvalue problem

$$G\,\gamma = \beta\, C\,\gamma \tag{31}$$

or, in partitioned form,

$$\left\{\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22}\end{bmatrix} - \beta\begin{bmatrix} I & 0 \\ 0 & 0\end{bmatrix}\right\}\begin{bmatrix}\gamma^{(1)} \\ \gamma^{(2)}\end{bmatrix} = \begin{bmatrix} 0 \\ 0\end{bmatrix} \tag{32}$$

for the minimum eigenvalues and the corresponding eigenvector. The above problem is equivalent to solving the pair

$$[(G_{11} - G_{12}G_{22}^{-1}G_{21}) - \beta I]\,\gamma^{(1)} = 0 \tag{33a}$$

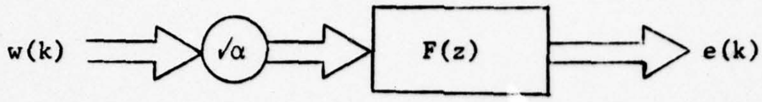
$$\gamma^{(2)} = -G_{22}^{-1}G_{21}\gamma^{(1)} \tag{33b}$$

The first part is solved as a usual eigenvalue problem. The eigenvector $\gamma^{(1)}$ corresponding to the minimum eigenvalue is selected and, then, from the second equation $\gamma^{(2)}$ is obtained. The desired parameter vector is finally obtained as

$$\hat{\gamma} = \frac{1}{\gamma^{(1)}(1)}\begin{bmatrix}\gamma^{(1)} \\ \gamma^{(2)}\end{bmatrix} \tag{34}$$

### 2.3.3 Additive Colored Noise in Output Signals

Suppose the error in output observations is assumed to be a stationary random process as shown in Figure 2, where $\sqrt{\alpha}$ is an unknown scalar, $F(z)$ a known (non-anticipatory) transfer function matrix and $w(k)$ a white noise vector sequence; that is $E\, w(k)w^T(\ell) = \delta_{k\ell} I$.




$$Z^{-1}\{F(z)\} = f(k)$$

Fig. 2. Error Process

Using f(k), the matrix of impulse responses of the entries of F(z), we can write

$$e(k) = \sqrt{\alpha} \sum_{\ell=0}^{\infty} f(\ell)\, w(k-\ell) \tag{35}$$

And, it can be shown easily that

$$E\, e(k)e^T(k+j) = \alpha \sum_{\ell=0}^{\infty} f(\ell) f^T(\ell+j) = \alpha R(j) \quad \text{(by definition)} \tag{36}$$

In the interest of avoiding degeneracy we assume that R(0) is positive definite.

We now return to the main problem of estimating $\gamma$. For convenience, let us rewrite equation (19),

$$S(k)\gamma = E(k)\gamma \tag{19}$$

Recall that only the first N+1 columns of E(k) are nonzero. In fact these columns are e(k), ...., e(k-N) respectively. As in subsection 2.3.2, we form the outer-product of (19) with itself and sum over k=0, ..,K. This yields an equation which is similar to (25) where, again, G and P are matrices given by (26) and (27) respectively. The difference arises in the fact that now

$$\begin{aligned} E\, P &= \alpha\, C \\ &= \alpha \begin{bmatrix} C^{(11)} & 0 \\ 0 & 0 \end{bmatrix} \end{aligned} \tag{37}$$

where the (N+1) x (N+1) matrix $C^{(11)}$ is given as

$$C^{(11)}_{ij} = \sum_{\ell=1}^{m} R_{\ell\ell}\,(|i-j|) \tag{38}$$

Note that this matrix is symmetric Toeplitz and nonsingular.

Following a procedure similar to that in subsection 4.2, it can be shown that the parameter vector $\gamma$ can be estimated by solving the pair

$$[C^{(11)^{-1}}(G_{11} - G_{12}G_{22}^{-1}G_{21}) - \alpha I]\,\gamma^{(1)} = 0 \tag{39a}$$

$$\gamma^{(2)} = -G_{22}^{-1}G_{21}\,\gamma^{(1)} \tag{39b}$$

The first part is solved as a usual eigenvalue problem. The eigenvector $\gamma^{(1)}$ corresponding to the minimum eigenvalue is selected and, then, from the second equation $\gamma^{(2)}$ is obtained. The desired parameter vector estimate is finally obtained as in equation (34) by adjoining these two parts and dividing by the first entry of the first part.

## 4.4 Additive White Noise in Both Input and Output Measurements

The measured input, u(k), is considered to be composed of measurement noise v(k) and the true input u(k)-v(k) see Fig. 3. The process v(k) is assumed to be zero mean and white. The model described in (11) must now be modified to

$$y(z) = H(z)[u(z) - v(z)] + e(z) \tag{40}$$

where the other symbols have their earlier defined meanings. Upon, multiplying this equation by A(z), the common denominator of the entries

(41)

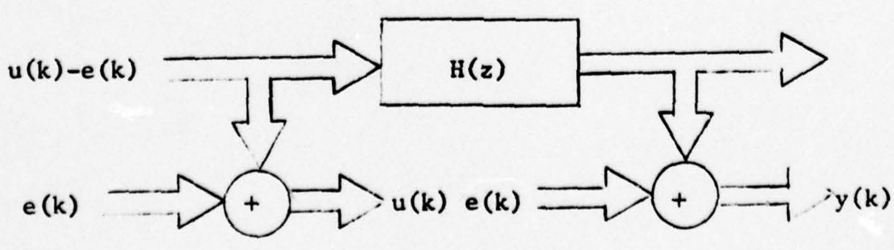



Fig. 3. Noisy input and output measurements

of H(z), one obtains

$$A(z)y(z) - B(z)u(z) - B(z)v(z) + A(z)e(z) \tag{41}$$

We now claim that (36) can be written in the time domain as

$$S(k)\gamma = E(k)\gamma \tag{42}$$

where S(k) is formed using y(k), ... ,y(k-N) and u(k), ... ,y(k-N) exactly as described in Section III. The matrix E(k) is, however, formed differently than the E(k) of equation (19). In fact, E(k) is now formed by replacing y(k) by e(k) and u(k) by v(k) in the matrix S(k). Thus whereas E(k) of (19) had a structure very different from that of S(k), the matrix E(k) of (42) has the same structure as S(k) except that for its entries the vectors e(k) and v(k) are used. Of course, e(k) and v(k) are only known statistically. They are assumed to be characterized by the correlations

$$\mathcal{E}\,[e(k)e^T(\ell)] = \delta_{k\ell}D \tag{43a}$$

$$\mathcal{E}\,[v(k)v^T(\ell)] = \delta_{k\ell}D' \tag{43b}$$

where D and D' are diagonal matrices of order m and r respectively. We shall denote their diagonal entries as $d_{ii}$ and $d'_{ii}$ respectively. Further, let d denote the trace of D.

Now, the outer product of (42) with itself when summed over k=0,1, ...,K yields

$$\gamma^T G\,\gamma = \gamma^T P\gamma \tag{44}$$

where

$$G = \sum_{k=0}^{K} S^T(k)S(k), \tag{45}$$

$$P = \sum_{k=0}^{K} E^T(k)E(k), \tag{46}$$

and

$$
E\,P = C = \begin{bmatrix} d\,I & & & & & \\ & d'_{11}I & & & & \\ & & d'_{22}I & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & d'_{rr}I \end{bmatrix} \tag{47}
$$

Each of the identity matrices in the above equation is of order $(N+1) \times (N+1)$. Following a procedure similar to that in the previous subsections, it can be shown that the parameter vector $\gamma$ can be estimated by solving the eigenvalue problem

$$
[C^{-1}\,G - \beta\,I]\gamma = 0 \tag{48}
$$

The eigenvector $\hat{\gamma}$ corresponding to the minimum eigenvalue is the desired estimate of the parameter vector.

## 2.4 MEASUREMENT FILTERS

The problem of identifying the desired parameter vector, $\gamma$ , was solved in the previous section via the key equation (19). The matrices S(k) and E(k) were formed by use of row vectors.

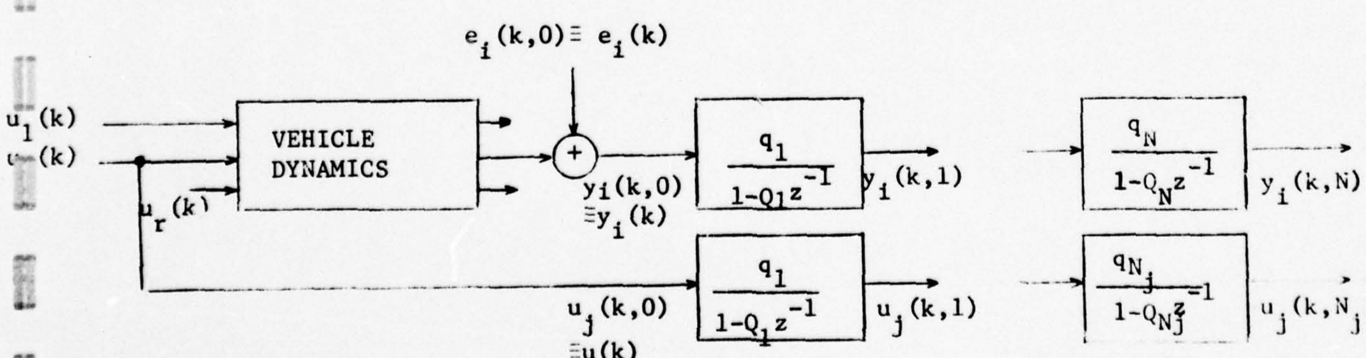



Fig. 4. Use of measurement filters to generate measurement sequences

$$
\begin{aligned}
y^{(i)}(k) &= [y_i(k), \ldots, y_i(k-N)] \\
u^{(j)}(k) &= [u_j(k), \ldots, u_j(k-N_j)] \\
e^{(i)}(k) &= [e_i(k), \ldots, e_i(k-N)]
\end{aligned} \tag{49}
$$

These vectors can be regarded as measurements resulting from a cascade of unit-delays ($z^{-1}$ each) processing the signals $y_i(k)$, $u_j(k)$, and $e_i(k)$. In practice, better identification results are achieved if these unit-delays are replaced by first order digital filters of the form $q_\ell/(1 - Q_\ell z^{-1})$, where $|Q_\ell| < 1$ for stability.

The precise arrangement is shown in Figure 4. Of course, the error signals $e_i(k)$, $i=0,..,m$ are not physically available. Rather, they are additively contained in $y_i(k)$, $i=0,..,m$ respectively. The signals resulting from this arrangement are denoted as $y_i(k,\ell)$, $u_j(k,\ell)$, and $e_i(k,\ell)$ ; $\ell=0,1, .. ,N$.

The matrices $S(k)$ and $E(k)$ are now formed by use of the row vectors

$$
\begin{aligned}
y^{(i)}(k) &= [y_i(k,0), \ldots, y_i(k,N)] \\
u^{(j)}(k) &= [u_j(k,N-N_j), \ldots, u_j(k,N_j)] \\
e^{(i)}(k) &= [e_i(k,0), \ldots, e_i(k,N)]
\end{aligned} \tag{50}
$$

exactly in the same fashion as they were formed by use of the vectors (35) in the earlier sections. The important fact concerning these new matrices is that the following relation holds:

$$S(k)\,\lambda = E(k)\,\lambda \tag{51}$$

where the vector $\lambda$ is related to the desired vector $\gamma$ in a known linear way. Specifically, it can be shown that

$$\gamma = \Gamma\,\lambda \tag{52}$$

where $\Gamma$ is a block diagonal matrix; that is

$$\Gamma = \text{diag}\,\{\Gamma_0, \Gamma_1, \ldots \Gamma_r, \Gamma_1, \ldots \Gamma_r, \ldots \Gamma_1, \ldots \Gamma_r\} \tag{53}$$

wherein each block $\Gamma_j$ is repeated m times. Further, $\Gamma_j$ is a $(N_j+1)\times(N_j+1)$ matrix whose $\mu, \nu$ entry can be generated by

$$\Gamma_{\mu-1,\nu} = \frac{1}{q_{\mu-N_j+N}} \left(\Gamma_{\mu,\nu} - Q_{\mu-N_j+N}\, \Gamma_{\mu,\nu-1}\right) \tag{54}$$

where

$$\Gamma_{N_j,\nu} = \begin{cases} 1 & \nu=1 \\ 0 & \nu>1 \end{cases} .$$

Each of the formulations in Section IV can be restated in terms of equation (51) and the corresponding G and P matrices with the following modifications:

- In place of vector $\gamma$, the vector $\lambda$ must appear in all of the equations in Section IV.
- The correlation matrix C ( in (31), (37) or (47)) must be replaced by the processed correlation matrix Z.
- After estimating $\lambda$, the estimate of the desired parameter vector $\gamma$ is computed as $\hat{\gamma} = \Gamma\hat{\lambda}$.

As to the matrix Z, for the formulation given in section 2.3.2 the $\mu,\nu$ entry of Z is given by

$$E \sum_{i=0}^{M-1} \left[ \sum_{j=0}^{i} e(i-j)\, h_\mu(j) \right] \left[ \sum_{k=0}^{i} e(i-k)\, h_\nu(k) \right] \tag{55}$$

where $h_\mu(k)$ is the impulse response of the filter

$$\prod_{\ell=1}^{\mu-1} \frac{q_\ell}{1 - Q_\ell\, z^{-1}} . \tag{56}$$

For uncorrelated noise the $\mu,\nu$ entry of Z is

$$\sum_{i=0}^{M-1} (M-i)\, h_\mu(i)\, h_\nu(i) . \tag{57}$$

Similar expressions for the processed correlation matrix Z can be derived for the cases in subsections 2.3.3 and 2.3.4. Indeed, for the former case the expression in (57) applies except that h (k) now represents the impulse response of the filter

$$\prod_{\ell=1}^{\mu-1} \frac{q_\ell}{1 - Q_\ell z^{-1}} \tag{58}$$

Nothing has been said so far concerning the choice of the parameters $q_\ell$ and $Q_\ell$ characterizing the measurement filters. Without loss of generality let us choose the gain parameter as $q_\ell = 1 - Q_\ell^2$. As for the choice of $Q_\ell$ we note that the measurement filters are low pass filters with cutoff frequencies $\frac{1}{T}$Ln $Q_\ell$ radian/second. They serve to decorrelate the measurement signals. When the input-output data is uncorrupted by noise, very accurate identification is achieved by a wide range of choice (over (0,1)) of the measurement filters. However, when the data is corrupted by noise, as must be the case in real flight test data, then the choice of $Q_\ell$ becomes the key to the success of accurate identification. Experience has shown that a set of filters with the cutoff frequencies suitably spaced within the approximate pass region of the system are highly effective. (For example. values that would attenuate the output signal(s) power evenly over 100 to 10% -- see Subroutine FINDQ). Often choosing all $Q_\ell$ to be identical is adequate.

## 2.5 STATE MODEL

It must first be emphasized that even though we had initially formulated the system in a state variable form, the identification was actually carried out in terms of a <u>transfer function matrix</u>. Let us denote the identified transfer function matrix as H(z) which when transformed to an equivalent s-domain form becomes H(s). The equivalence may be impulse, pulse or

trapezoidal equivalence. When the sampling rate is adequately high each of these equivalence transformations yields approximately the same matrix H(s).

Now, a state model can be obtained by the procedure discussed below. It is assumed for simplicity, and no loss of generality in a practical sense, that the poles of H(s) are simple. Thus H(s) has the form

$$H(s) = \frac{1}{\prod_{\ell-1}^{N} (s+\alpha_\ell)} [B_{ij}(s)] \tag{59}$$

where $B_{ij}(s)$ are the numerator polynomials. A partial fraction expansion yields

$$H(s) = R^0 + \sum_{i=N}^{N} R^i \frac{1}{s+\alpha_i} \tag{60}$$

The residue matrices $R^i$ have the same dimensionality as H(s). They are m x r each. Further, these residue matrices $R^i$ can be decomposed as

$$R^i = \tilde{R}^i L^i \tag{61}$$

where $\tilde{R}^i$ is a matrix whose columns are simply a maximal set of linearly independent columns of $R^i$. It can be shown that if the system was identified perfectly, i.e., if the identified matrix H(s) had coincided with the true transfer function matrix, then each of the matrices $\tilde{R}^i$ would have a rank equal to 1. In practice, this will not exactly be the case, particularly if one or more of the system modes does not appear strongly in any of the outputs. In reference [2] such a system was called as ill posed.

Let us assume that the system is well-posed so that each of the matrices $\tilde{R}^i$ has an essential rank equal to 1. A matrix will be said to have an essential rank equal to 1 if its column (or row) vectors lie in a two-sided poly-cone of sufficiently small angle. The analyst must decide for himself if the angle of the two sided poly-cone in a particular case is small enough. Assuming that this has been done one determines the decomposition (61) such that $\tilde{R}^i$ is a column vector and the error is minimized in some sense. For example, $\tilde{R}^i$ could be taken to be the median of the poly-cone (see Fig. 5).

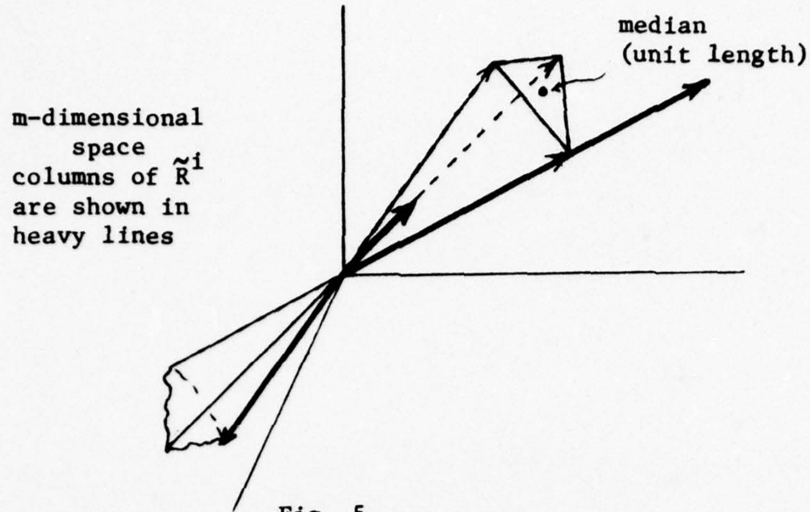



Fig. 5

*Note that since* $\tilde{R}^i$ *is a column vector,* $L^i$ *is a row vector.* Then the state model can be written as

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & & \\ & \alpha_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \alpha_N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_N \end{bmatrix} + \begin{bmatrix} L^1 \\ L^2 \\ \cdot \\ \cdot \\ L^N \end{bmatrix} u \tag{62a}$$

$$y = [\tilde{R}^1 \quad \tilde{R}^2 \; . \; . \quad \tilde{R}^N] \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_N \end{bmatrix} + R^0 \quad u \tag{62b}$$

A simple example is presented to clarify the procedure. Consider that as a result of the identification procedure the transfer function matrix of a two-input, two-output system is identified as

$$H(s) = \frac{1}{(s+1)(s+2)(s+3)} \begin{bmatrix} s^2+s+0.95 & s^2+4.8 \\ 0.97 & s^2+5.9s+10.5 \end{bmatrix}$$

$$= \frac{1}{s+1}\begin{bmatrix} 0.475 & 2.9 \\ 0.485 & 2.8 \end{bmatrix} + \frac{1}{s+2}\begin{bmatrix} -2.95 & -8.8 \\ -0.97 & -2.7 \end{bmatrix} + \frac{1}{s+3}\begin{bmatrix} 3.475 & 6.9 \\ 0.485 & 0.9 \end{bmatrix}$$

$$\simeq \frac{1}{s+1}\begin{bmatrix}0.7096\\ 0.7046\end{bmatrix}[0.6789\ \ 4.0311] + \frac{1}{s+2}\begin{bmatrix}0.9530\\ 0.3029\end{bmatrix}[-3.1054\ \ -9.2049]$$

$$+ \frac{1}{s+3}\begin{bmatrix}0.9910\\ 0.1338\end{bmatrix}[3.5087\ \ 6.9584]$$

Note that the poly-cone angles for the three residue matrices were $1.6^{\circ}$, $1.14^{\circ}$ and $0.51^{\circ}$. The state model can be written as

$$\frac{d}{dt}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}-1 & & \\ & -2 & \\ & & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0.6789 & 4.0311\\ -3.1054 & -9.2049\\ 3.5087 & 6.9584\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}0.7096 & 0.9530 & 0.9910\\ 0.7046 & 0.3029 & 0.1338\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

It should be pointed out that if a pair of eigenvalues turns out to be complex conjugate pair, then the entries in the matrices of the state model will appear in complex conjugate rows. Further, the corresponding state variables will also be complex valued, even though the output vector y will turn out to be real. Such a state model can be transformed into an equivalent real state model by use of the conversion matrices

$$J = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ j & -j\end{bmatrix}, \qquad J^{-1} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & -j\\ 1 & j\end{bmatrix}$$

Consider, specifically, that the first two eigenvalues in (62a) are complex (and conjugates of each other). Then the equations (62a) and (62b), i.e., $\dot{x} = F^1x + F^2u$, $y = F^3x + F^4u$ can be transformed into a real state model by defining

$$\chi = P\,x \tag{63}$$

where P is a matrix whose top left hand corner is the matrix J, the remaining diagonal entries are unity and all other entries are zero. The real state model becomes

$$\begin{aligned} \dot{\chi} &= (PF^1P^{-1})\chi + (PF^2)u \\ y &= (F^3P^{-1})\chi + F^4u \end{aligned} \tag{64}$$

## 2.6 APPLICATION TO SIMULATED FLIGHT TEST DATA OF A SIX MAN SUBMERSIBLE

The longitudinal dynamics of a six man submersible was simulated and studied with the objective of identifying its transfer functions under non-zero initial conditions. The transfer function matrix, as determined by the RGEORGE computer program from vehicle geometry and supplied to us by the NCSL, was

$$H(s) = \begin{bmatrix} u(s)/\delta_s(s) \\ w(s)/\delta_s(s) \\ \theta(s)/\delta_s(s) \end{bmatrix} \quad \begin{matrix} \text{ft./sec./degree} \\ \text{ft./sec./degree} \\ \text{dimensionless} \end{matrix}$$

$$= \frac{1}{D(s)} \begin{bmatrix} -0.001973s^3-0.002559s^2+0.00003455s-0.00003706 \\ -0.010383s^3-0.031274s^2-0.0025350s-0.00004588 \\ -0.3432s^2-0.173840s-0.0086310 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{(-0.001973)(s-0.012012 \pm j\,0.11863)(s+1.3208)}{(s+0.0091898 \pm j\,0.11378)(s+0.055798)(s+1.4057)} \\[2ex] \dfrac{(0.010383)(s+0.027043)(s+2.9291)}{(s+0.0091898 \pm j\,0.11378)(s+1.4057)} \\[2ex] \dfrac{(-0.3432)(s+0.45072)}{(s+0.0091898 \pm j\,0.11378)(s+1.4057)} \end{bmatrix}$$

where the common denominator polynomial D(s) is

$$\begin{aligned} D(s) &= s^4 + 1.47987s^3 + 0.11833s^2 + 0.020484s + 0.00102196 \\ &= (s + 0.0091898 \pm j\,0.11378)(s + 0.055798)(s + 1.4057) \end{aligned}$$

In the simulation studies performed, a sampling interval of 0.5 sec. was used. The equivalent z-domain transfer function matrix, obtained on a pulse invariance basis [6],[7], is

$$H(z) = \frac{z^{-1}}{D(z)} \begin{bmatrix} -0.00094609+0.0023897z^{-1}-0.0019407z^{-2}+0.0004954z^{-3} \\ -0.0067869 +0.014612z^{-1} -0.0090902z^{-2}+0.0012629z^{-3} \\ -0.036992 +0.033999z^{-1} +0.027069z^{-2} -0.024456z^{-3} \end{bmatrix}$$

where

$$D(z) = 1 -3.45527z^{-1} +4.38953z^{-2} -2.41136z^{-3} +4.77143z^{-4}$$

The response of the vehicle to a unit pulse stern-plane input, computed by use of $H(z)$ above, is shown in Figure 6. Note that the short period transient governed by the mode $\exp(-1.4057t)$ manifests itself only in the forward velocity $u(t)$. The other modes manifest in each of the three longitudinal variables $u$, $w$ and $\theta$. The problem of identification is therefore marginally well-posed, or perhaps ill-posed, for fourth order identification.

Noise-free study

Three different runs were made on the computer program MGRAM (multivariable Gram program with initial conditions capability).

Run 1: Non-zero initial conditions, with zero stern plane input. Specifically,

$$u(0) = -0.018$$

$$w(0) = -0.022$$

$$\theta(0) = -4.31$$

$$\dot{\theta}(0) = 0.0$$

The response is shown in Fig. 7. The system, i.e., the initial conditions part $h(z)$ in (8) (or its s-domain equivalent), was identified with insignificant error. Hence, the details of identification are not presented (they are presented for the more interesting, noisy data case). In reconstruction, the ratio

of rms error to rms signal were

0.00001% for u

0.00001% for w

0.00001% for $\theta$

Run 2: Zero initial conditions were assumed and a stern plane input

$$\delta_s(t) = \begin{cases} 0.5^\circ & \text{for } 0 \le t \le 62.5 \text{ sec} \\ -0.5^\circ & \text{for } 62.5 < t \le 125.0 \text{ sec} \\ 0 & \text{for } 125 < t \le 250 \text{ sec} \end{cases}$$

was applied. The response is shown in Fig. 8. Again the system was identified almost perfectly. Rms error to rms. signal ratios turned out to be

0.00001% for u

0.00001% for w

0.00001% for $\theta$

The details are omitted (the more interesting, noisy data case is presented later in detail)

Run 3: Nonzero initial conditions as in Run 1 and a stern plane input as in Run 2 were used. The response is shown in Fig. 9. Note that the response in this case is the sum of the responses in Figures 7 and 8, since the system is linear. As might be expected, the system was identified with insignificant errors. The identified system appeared in the form

$$[H^1(z) \,\vdots\, h(z)]$$

or the equivalent s-domain form

$$[H^1(s) \,\vdots\, h(s)]$$

where $H^1(s)$ is the part identified in Run 2, while h(s) is the initial conditions part identified in Run 1.

Noisy-Data Study

The dynamics of the six man submersible is now studied using corrupted data. Each variable was corrupted with white noise such that the ratio of rms values of the noise to signal was 5%. Three different runs were made on the computer program MGRAM (multi-variable Gram program with initial conditions capability).

Run 4: The non-zero initial conditions were the same as in the noise-free Run 1. Stern plane input was identically zero. To each of the responses an independent white noise sequence was added (rms ratio = 5%). Compare Fig. 10 with Fig. 7. The results of identification are presented in Table 2.1 and in Figure 11. The latter depicts the reconstructed outputs, comparing them to the true (uncorrupted) responses. It should be emphasized that the matrix identified here forms the second half of the matrix

$$[H^1(s) \mid h(s)]$$

where the first half denotes the transfer matrix between the stern plane input and the longitudinal variables. The errors in reconstruction are

0.60% for $u$

0.89% for $w$

0.28% for $\theta$

Run 5: Zero initial conditions were assumed and a stern plane input as in Run 2 was applied. The corrupted response ( 5% noise) is shown in Fig. 12. Compare Fig. 12 with Fig. 8. The results

of identification are given in Table 2.2 and Fig. 13. The latter shows the reconstructed outputs together with the true (uncorrupted) responses. Note that the matrix identified here was the input-output matrix $H^1(s)$. The errors in reconstruction are

0.86% for u

1.06% for w

0.37% for θ

Run 6: Nonzero initial conditions as in Run 1 (or Run 4) and a stern plane input as in Run 2 (or Run 5) were used. The response corrupted by noise (5% on each variable) is shown in Fig. 14. The results of identification by the computer program MCRAM are given Table 2.3 and Fig.15. The latter compares the reconstructed outputs with the true outputs. Both matrices $H^1(s)$, *representing the transfer matrix*, and h(s), arising from the initial conditions, were determined. The errors in reconstruction are

0.98% for u

1.31% for w

0.51% for θ

Discussion: In each of the three noisy runs (Runs 4, 5 and 6) the weak mode (exp(-1.4057t)), present only the data of the variable u, is missed. However the other modes - - poles as well as the associated weighting factors - - are identified quite accurately. This is borne out by the rms errors in reconstruction, which was less than 1.5% in every case. The defection of the micromode could be improved by (a) reducing the sampling interval and (b) incorporating a slow exponential phase in the input, possibly following the doublet.

TABLE 2.1

Results of Longitudinal Dynamics Identification of a Six Man Submersible

Non-zero Initial conditions: $u(0) = 0.018$, $w(0) = 0.022$, $\theta(0) = 0.0$
Stern plane Input $\delta_s(t) = 0$

Ratio of Noise to Signal rms Values = 5% for each output variable.
Q Parameters: 0.890625, 0.919922, 0.943359, 0.971680

---

Identified transfer functions

z-domain

$$h(z) = \frac{1}{D(z)} \begin{bmatrix} -0.0185 & +0.0539z^{-1} & -0.0525z^{-2} & +0.0170z^{-3} \\ -0.0262 & +0.0812z^{-1} & -0.0832z^{-2} & +0.0283z^{-3} \\ -4.33 & +12.6z^{-1} & -12.2z^{-2} & +3.92z^{-3} \end{bmatrix}$$

$$D(z) = 1-3.89344z^{-1} +5.68655z^{-2} -3.69244z^{-3} +.899341z^{-4}$$

s-domain

$$h(s) = \frac{1}{D(s)} \begin{bmatrix} -0.0369s^{3} & -0.00589s^{2} & -0.000684 & -0.0000772 \\ -0.0524s^{3} & +0.0101s^{2} & +0.00370s & +0.000157 \\ -8.66s^{3} & -1.68s^{2} & -0.0636s & +0.000106 \end{bmatrix}$$

$$D(s) = s^{4} +.212187s^{3} +0.0241950s^{2} +0.002665745s +0.0000989037$$

$$= (s +0.0099371 \pm 0.0566106\, j)\,(s + 0.0544683)\,(s + 0.139284)$$

Reconstruction Errors: 0.60% for u
0.89% for w
0.28% for $\theta$

TABLE 2.2

Results of Longitudinal Dynamics Identification of a Six Man Submersible

Non-zero Initial conditions: $u(0) = 0.0$, $w(0) = 0.0$, $\theta(0) = 0.0$, $\dot{\theta} = 0.0$

Stern plane Input $\delta_s(t)$ = doublet, duration = 125 sec., amplitude = 0.5 degree

Ratio of Noise to Signal rms Values = 5% for each output variable.

Q Parameters: 0.933594, 0.953124, 0.970703, 0.983643

---

Identified transfer functions

<u>z-domain</u>

$$h(z) = \frac{1}{D(z)} \begin{bmatrix} -0.00125z^{-1} & +0.00248z^{-2} & -0.00122z^{-3} & -0.0000231z^{-4} \\ -0.000585z^{-1} & -0.0140z^{-2} & +0.0291z^{-3} & -0.0145z^{-4} \\ 0.0560z^{-1} & -0.330z^{-2} & +0.489z^{-3} & -0.175z^{-4} \end{bmatrix}$$

$$D(z) = 1 - 2.51950z^{-1} + 1.61875z^{-2} + 0.326244z^{-3} - 0.425364z^{-4}$$

<u>s-domain</u>

$$h(s) = \frac{1}{D(s)} \begin{bmatrix} -0.00221s^{3} & -0.00290s^{2} & +0.0000396s & -0.0000425 \\ -0.0100s^{3} & -0.0366s^{2} & -0.00291s & -0.0000533 \\ 0.00783s^{3} & -0.377s^{2} & -0.200s & -0.00982 \end{bmatrix}$$

$$D(s) = s^4 + 1.70962s^3 + 0.133473s^2 + 0.0236459s + 0.00116332$$

$$= (s + 0.00917945 \pm 0.113694\, j)\,(s + 0.0546324)\,(s + 1.63663)$$

Reconstruction Errors: 0.86% for u

1.06% for w

0.37% for θ

TABLE 2.3

Results of Longitudinal Dynamics Identification of a Six Man Submersible

Non-zero Initial conditions: $u(0) = -.018$, $w(0) = -.022$, $\theta(0) = -4.31$,
$\dot{\theta} = 0.0$ Stern plane Input $\delta_s(t)$ = Doublet, duration=125sec, amplitude=0.5 degree

Ratio of Noise to Signal rms Values = 5% for each output variable.
Q Parameters: 0.931641, 0.953125, 0.970703, .984131

---

Identified transfer functions

<u>z-domain</u> $h(z) =$

$$\frac{1}{D(z)}\left[\begin{array}{c|c} (-0.00146z^{-1} +0.00321z^{-2} -0.00202z^{-3} +0.000275z^{-4}) & (-0.0175 +0.0283z^{-1} -0.00457z^{-2} -0.00633z^{-3}) \\ (0.000371z^{-1} -0.0159z^{-2} +0.0300z^{-3} -0.0145z^{-4}) & (-0.0179 +0.0179z^{-1} -0.0243\, z^{-2} -0.0240\, s^{-3}) \\ (0.0719z^{-1} -0.369z^{-2} +0.482z^{-3} -0.186z^{-4}) & (-4.11 +6.34z^{-1} -0.520z^{-2} -1.71z^{-3}) \end{array}\right]$$

$$D(z) = 1 -2.61323z^{-1} +1.89615z^{-2} +0.0522966z^{-3} -0.335096z^{-4}$$

<u>s-domain</u> $h(s)=$

$$\frac{1}{D(s)}\left[\begin{array}{c|c} (-0.00271s^3 -.00376s^2 +0.0000535s -0.0000547) & (-0.0350s^3 -0.0758s^2 -0.00238s -0.00115) \\ (-0.00408s^3 -0.0467s^2 -0.00371s -0.0000682) & (-0.0348s^3 -0.108s^2 +0.0402s +0.00238) \\ (0.0980s^3 -0.453s^2 -0.257s -0.0126) & (-8.21s^3 -18.8s^2 -0.986 +0.00929) \end{array}\right]$$

$$D(s) = s^4 +2.18668s^3 +0.167964s^2 +0.0303190s +0.00149231$$
$$= (s +0.00926847 \pm 0.113652\, j)\,(s + 0.0542944)\,(s + 2.11384)$$

Reconstruction Errors:

0.98% for u

1.31% for w

0.51% for $\theta$

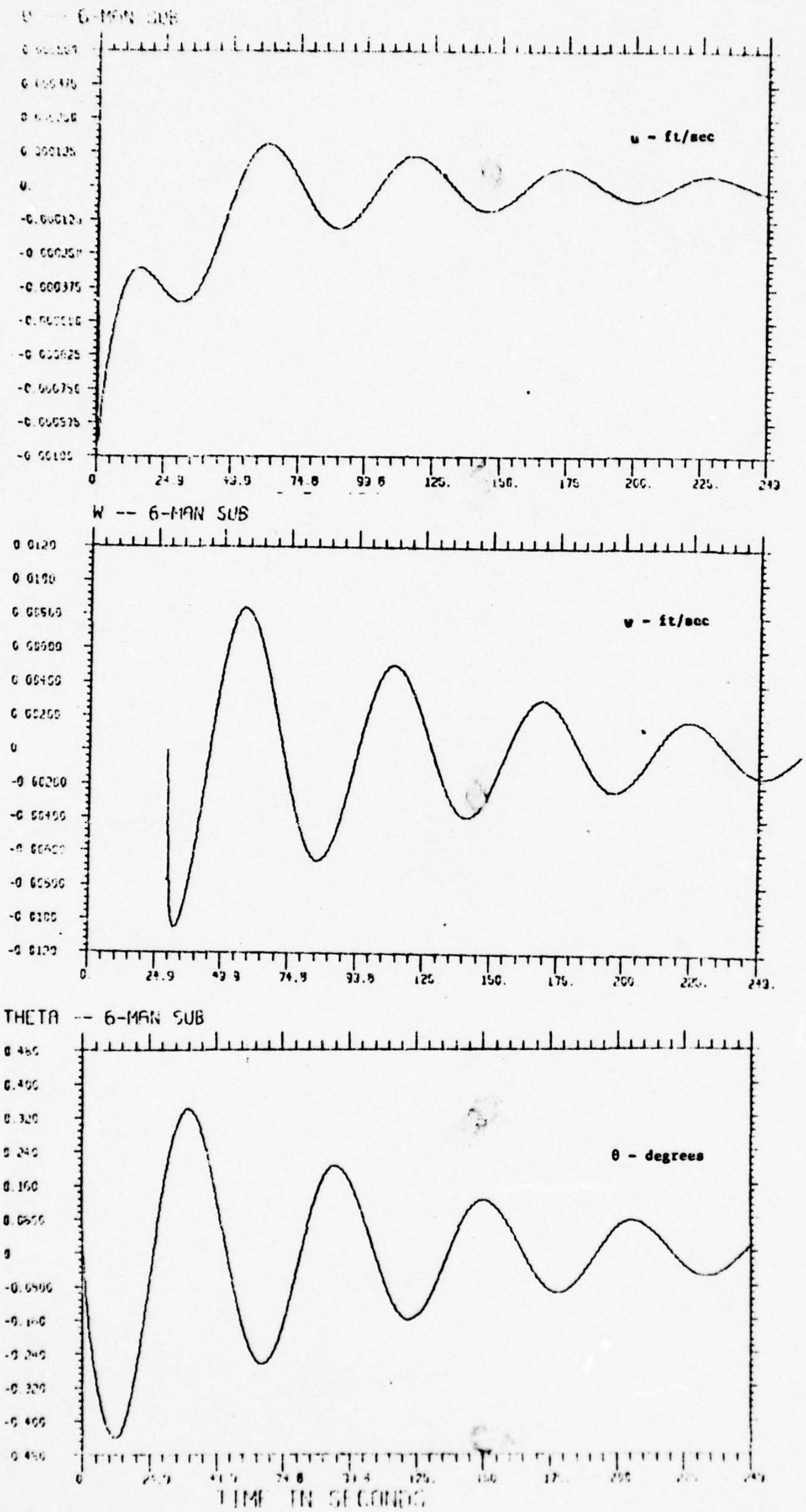



Fig. 6. Vehicle response to a unit impulse stern plane input.

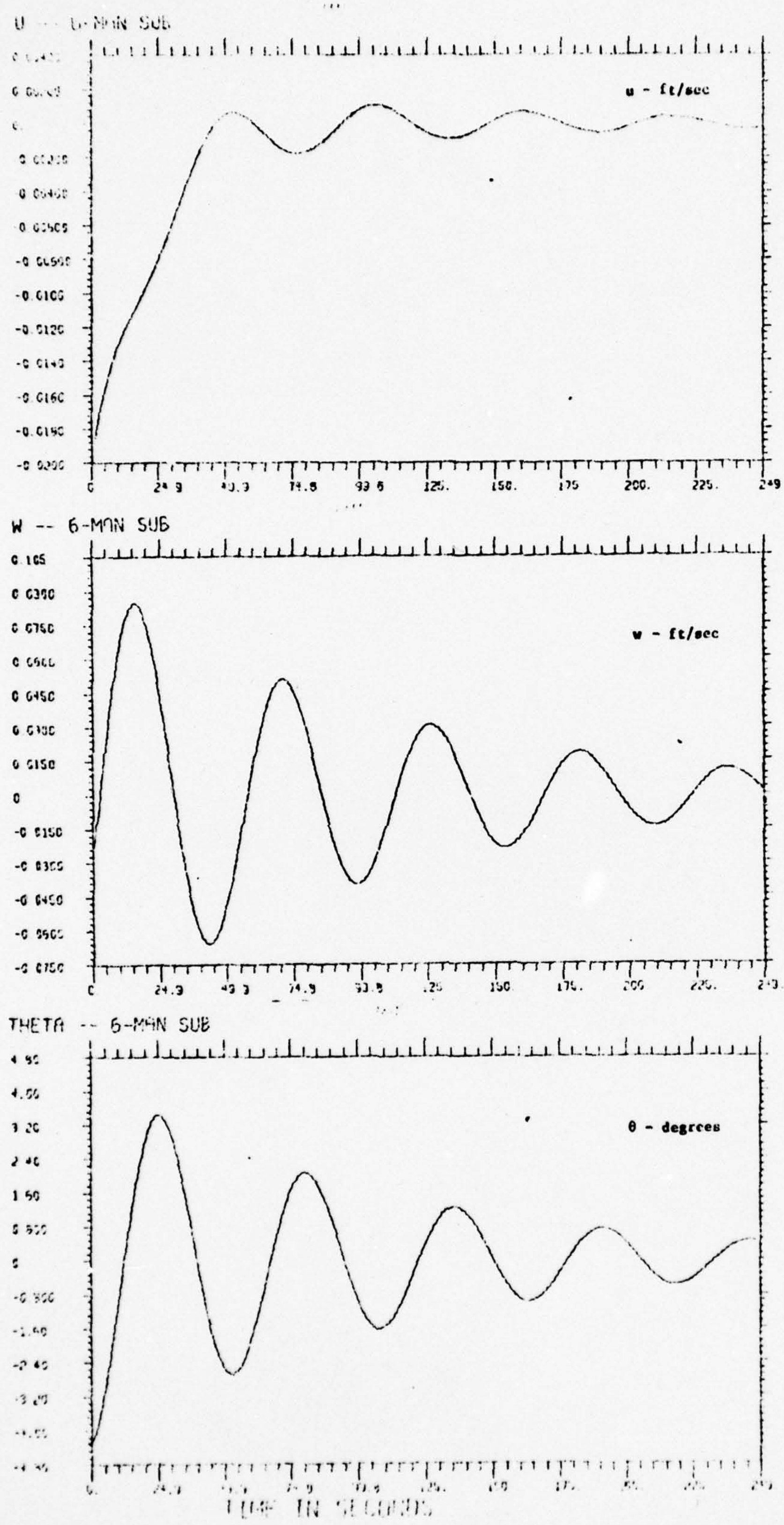



Fig. 7. Vehicle response to a non-zero initial conditions; $\delta_s(t)$ 0 (Run 1)

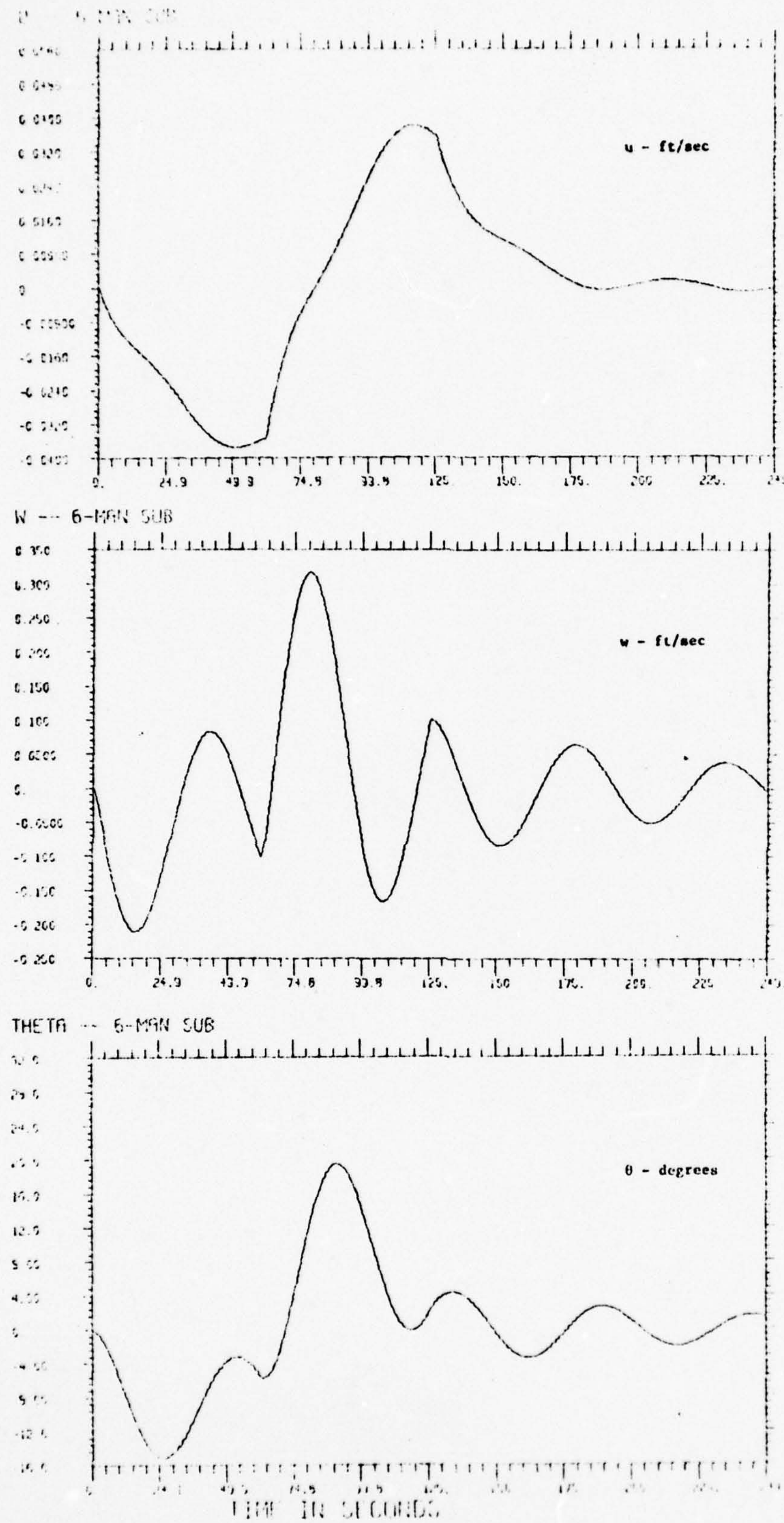



Fig. 8. Vehicle response to a stern-plane input and zero initial conditions (Run 2)

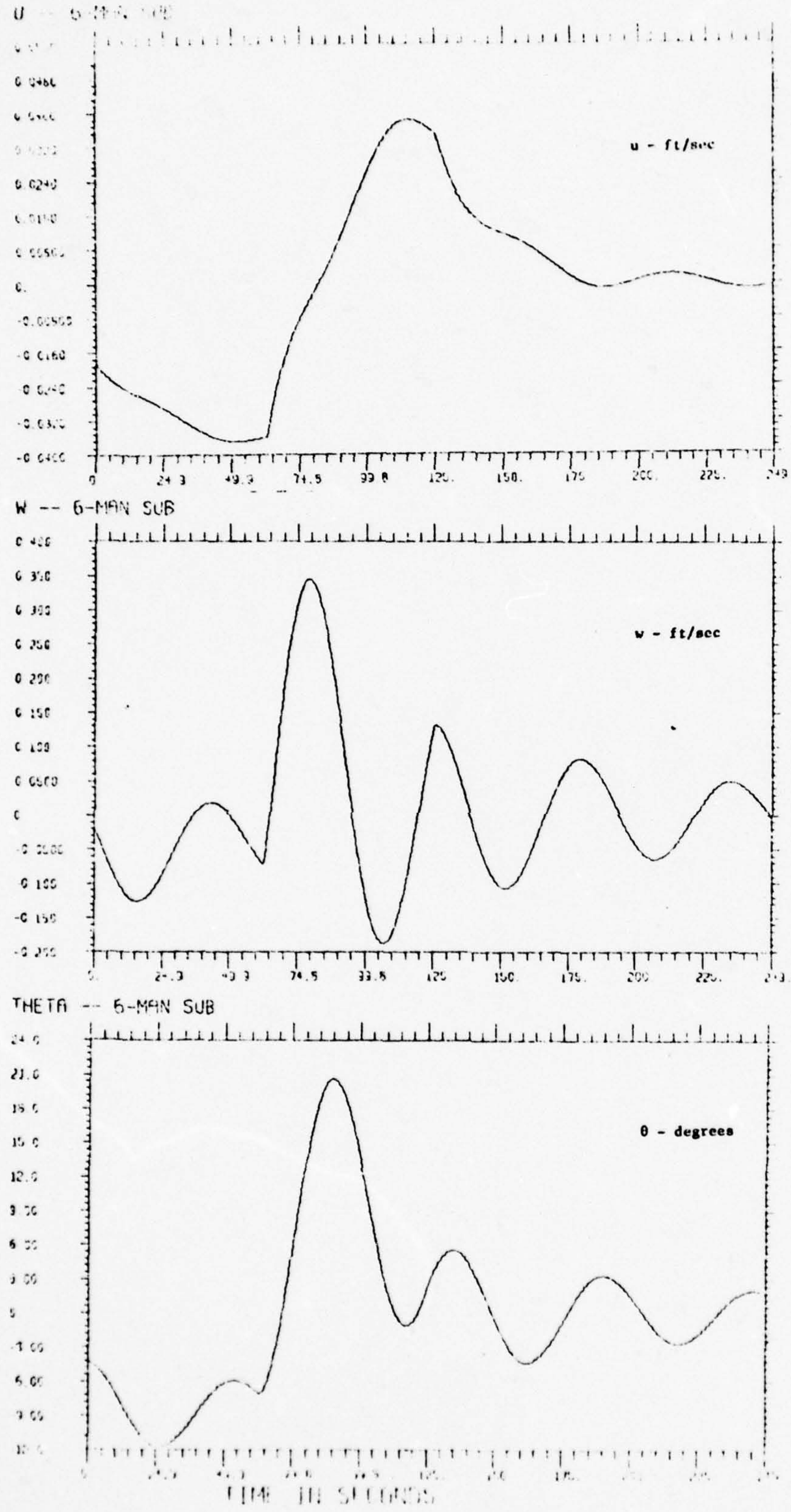



Fig. 9. Vehicle response to a stern-plane input and non-zero initial conditions (Run 3)

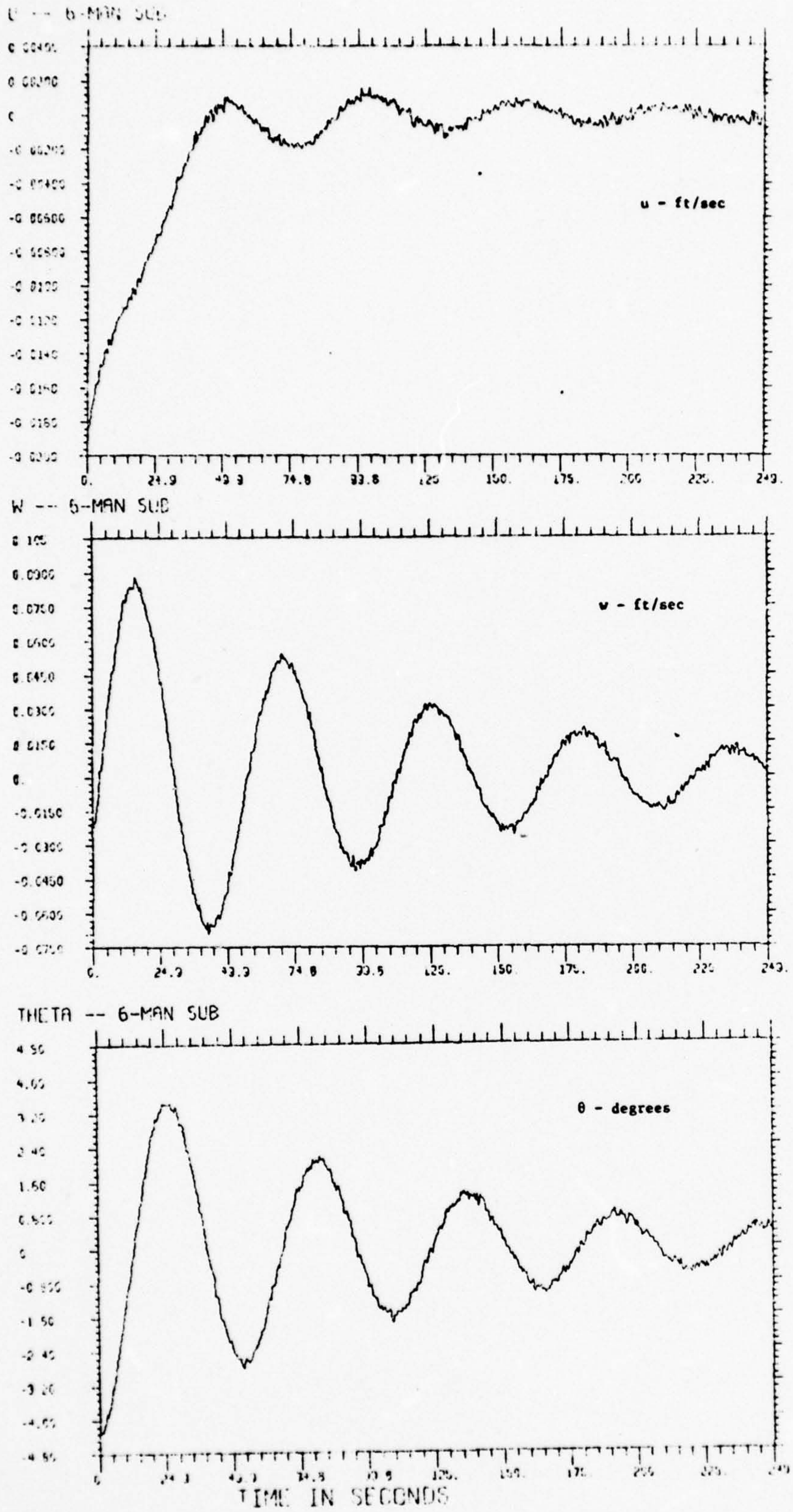



**Fig. 10. Noisy measurement of vehicle response. Non-zero initial conditions, $\delta_s(t)$ 0 (Run 4)**

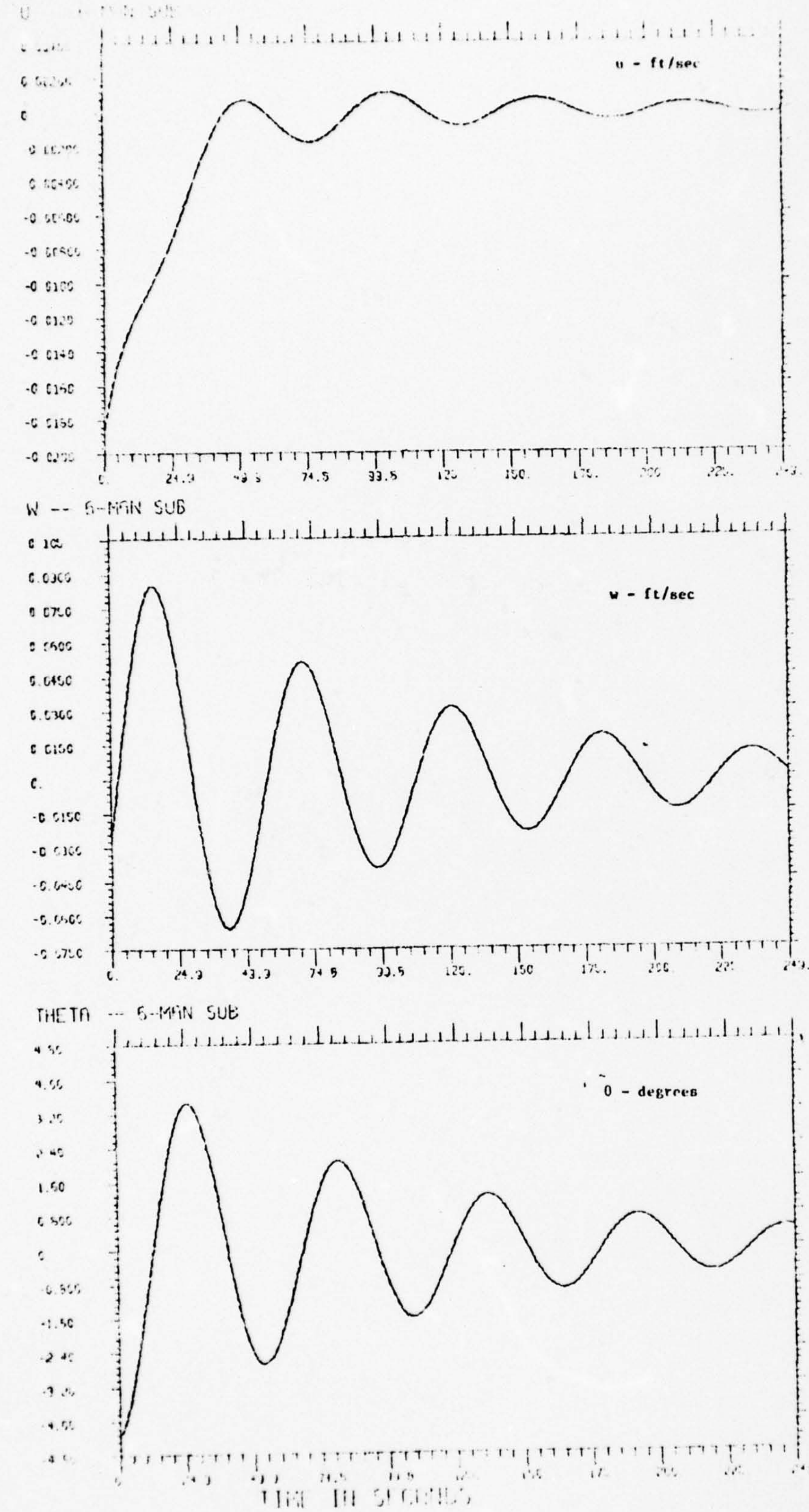



Fig. 11. Reconstructed outputs vs. true vehicle response. Non-zero initial conditions, $_{s}(t)$ 6 (Run 4)

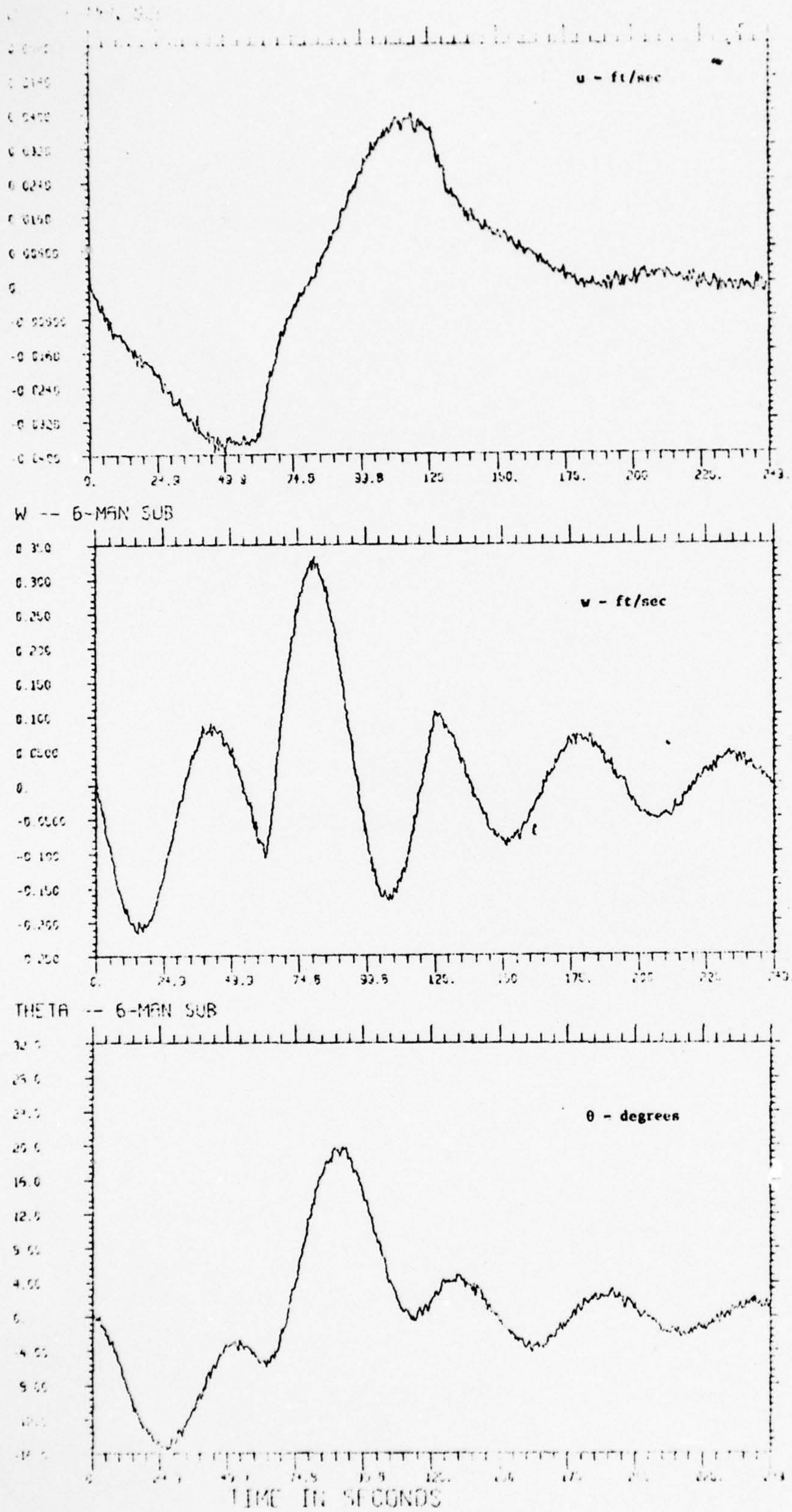



Fig. 12. Noisy measurement of vehicle response to a stern-plane input and zero initial conditions (Run 5)

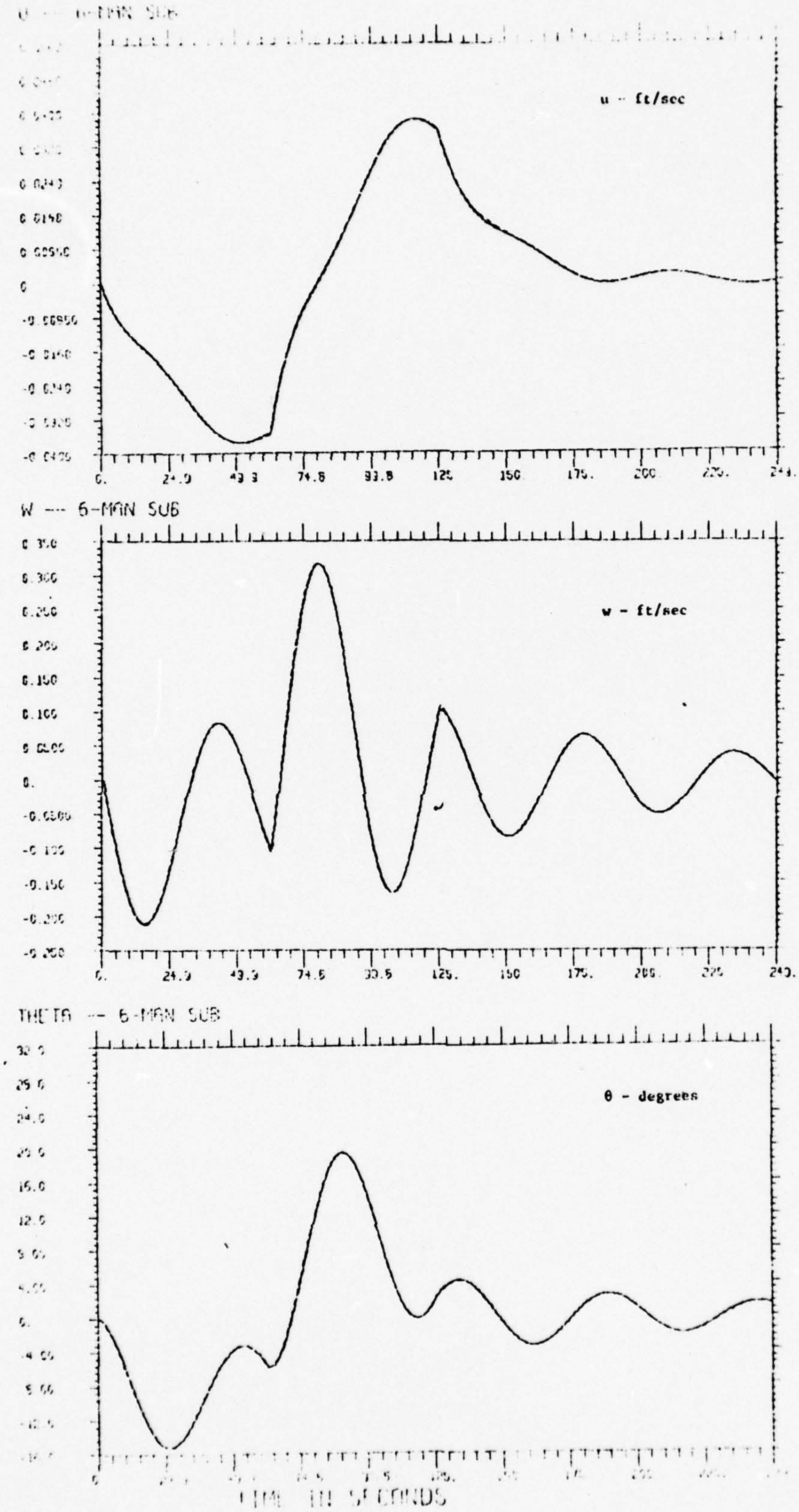



Fig. 13. Reconstructed outputs vs. true vehicle response to a sternplane input and zero initial conditions (Run 5)

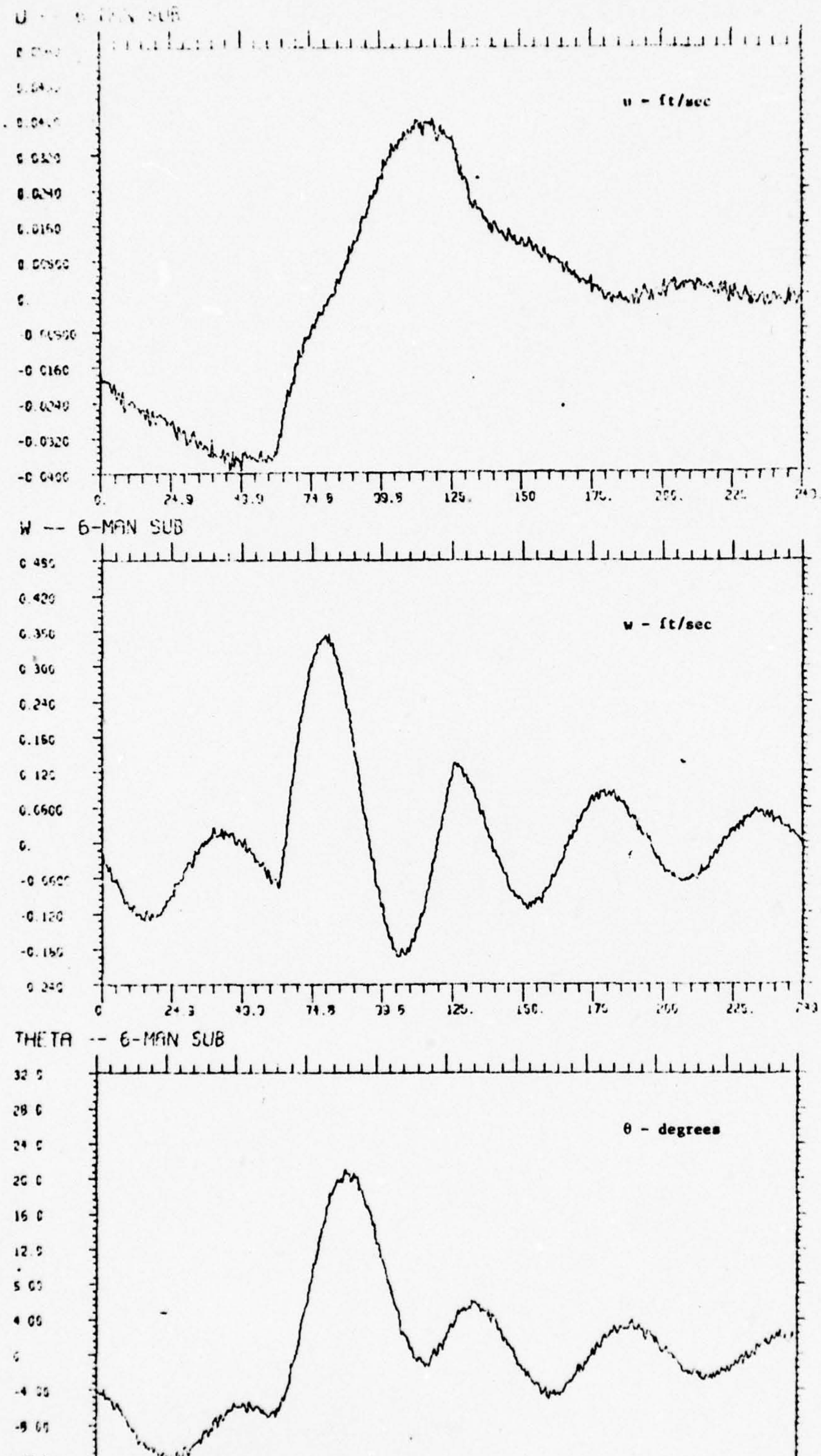



Fig. 14. Noisy measurement of vehicle response to a stern-plane input and non-zero initial conditions (Run 6)

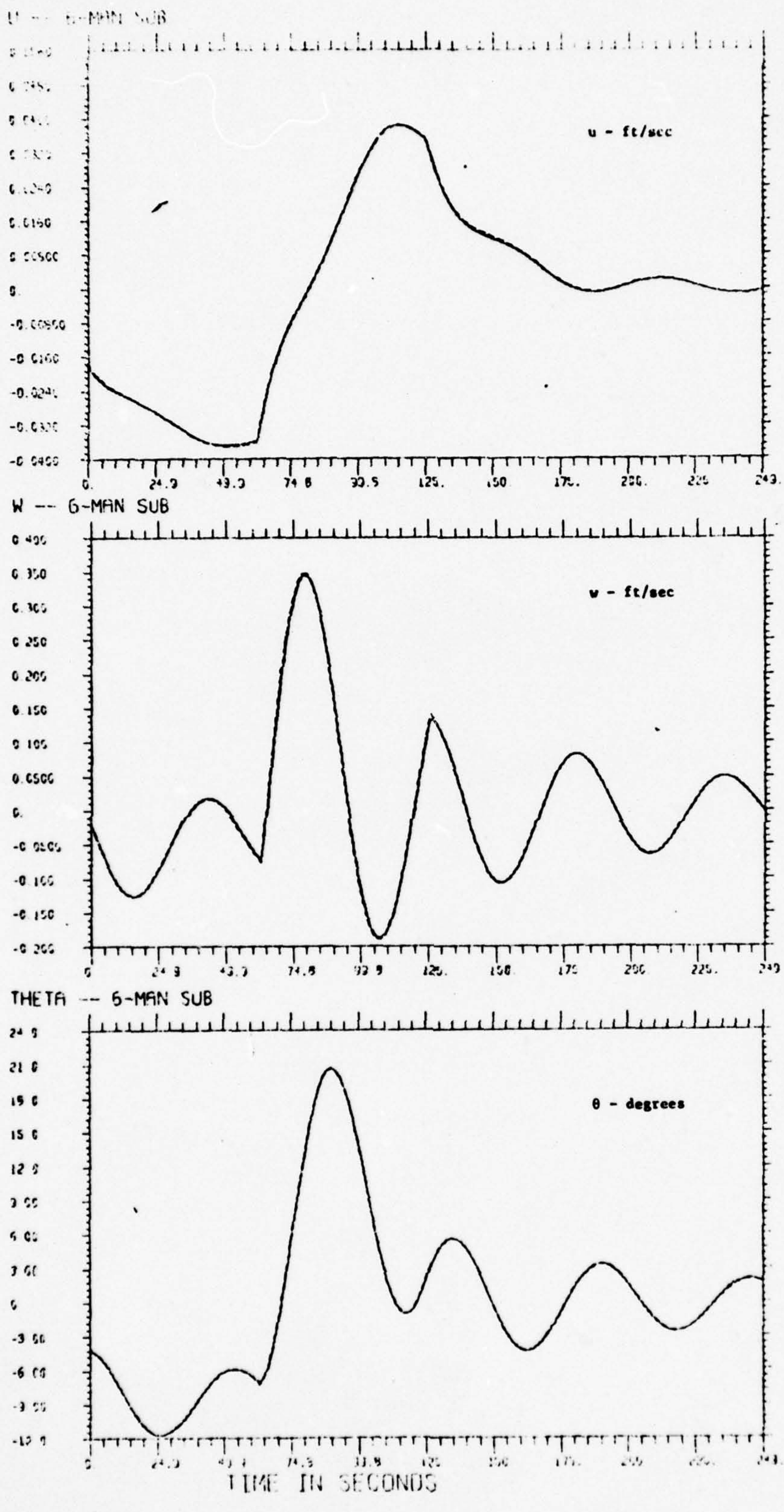



**Fig. 15. Reconstructed outputs vs. true vehicle response to a stern-plane input and non-zero initial conditions (Run 6)**

## 2.7 DETERMINATION OF INITIAL STATE

In the previous sections it has been shown that the vehicle dynamics can be identified successfully in the form

$$H(z) = [H^1(z) \mid h(z)]$$

or the equivalent s-domain form

$$H(s) = [H^1(s) \mid h(s)] \tag{65}$$

Here, $H^1(s)$ is the input-output traksfer matrix and $h(s)$ is the contribution of the initial conditions. In fact, if the continuous-time state model of the vehicle were

$$\dot{x} = G^1 x + G^2 u(t) \tag{66a}$$

$$y = G^3 x + G^4 u(t) \tag{66b}$$

then analogous to equations (7) and (8) we would have

$$H^1(s) = G^3(sI - G^1)^{-1}G^2 + G^4 \tag{67}$$

$$h(s) = G^3(sI - G^1)^{-1}x(0) \tag{68}$$

Problem Statement

Given $H^1(s)$ and $h(s)$, obtain the initial state for the state model (66) (derived from $H^1(s)$ via the procedure of Section 2.5).

Solution

Using the notation of Section 2.5,

$$H^1(s) = \sum_{i=1}^{N} \tilde{R}^i L^i \left(\frac{1}{s+\alpha_i}\right) + R^0$$

where the column vectors $\tilde{R}^i$, the row vectors $L^i$, the matrix $R^0$ and the poles $-\alpha_i$ are all known. Since $G^4 = R^0$ and

$$G^2 = \begin{bmatrix} L^1 \\ L^2 \\ \cdot \\ L^n \end{bmatrix}$$

in the state model developed in Section 2.5, we deduce that

$$G^3(sI - G^1)^{-1} = \left[\frac{1}{s+\alpha_1} \tilde{R}^1 \quad . \; . \; . \; . \quad \frac{1}{s+\alpha_n} \tilde{R}^N\right]$$

Substitution of this relation into (68) yields

$$h(s) = [\frac{1}{s+\alpha_1} \tilde{R}^1 \; . \; . \; . \; . \; \frac{1}{s+\alpha_n} \tilde{R}^N] \; x(0)$$

$$= \sum_{i=1}^{N} \frac{1}{s+\alpha_i} \; x_i(0) \; \tilde{R}^i$$

A partial fraction expansion of the known h(s) and comparison of $\frac{1}{s+\alpha_i}$ terms with the right hand side in (69) will yield the initial state x(0). In fact, the initial state may be determined from a single row of equation (69), for example $h_3(s) = h_\theta(s)$ row, provided the state is observable in the variable corresponding to that row. The latter is equivalent to requiring nonzero entries in the vectors $\tilde{R}^i$ in that particular row. For the sake of illustration let us assume that such is the case for the third variable. Then we have

$$h_3(s) = \sum_{i=i}^{N} \frac{c_{3i}}{s+\alpha_i} = \sum_{i=1}^{n} \frac{1}{s+\alpha_i} \; x_i(0) \; \tilde{r}_3^{\,i}$$

so that

$$x_i(0) = \frac{c_{3i}}{\tilde{r}_3^{\,i}}$$

<u>Application</u>

The importance of the above algorithm arises in the following way. For definiteness only the case of longitudinal dynamics is discussed.

Suppose the vehicle dynamics has been identified during the initial tests as

$$\dot{x} = G^1 x + G^2 u, \qquad y = G^3 x + G^4 u$$

where $y = (u, w, \theta)$ and $u = (\delta_s, \delta_b)$. In the case of a towed vehicle there will be additional inputs. It should be clear that this identification will require measurement of each of the five variables in y and u. However, such complete measurements will be required one time only, during the initial tests.

Let us now suppose that during normal operation of the vehicle only the pitch angle and the input deflections are measured. Then using the procedure discussed above we can determine the state of the system x(0) and therefrom

$$y(0) = \begin{bmatrix} u(0) \\ w(0) \\ \theta(0) \end{bmatrix} = G^3 x(0)$$

since $G^3$ is known. Further,

$$\frac{d}{dt} y(0) = G^3[G^1 x(0) + G^2 u(0)]$$

can also be found. To summarize:

<u>If the system dynamics has been identified completely in the initial tests</u> then it is possible, in principle, to determine the variables u(t), w(t) and θ(t) as well as their derivatives by measuring only the pitch angle during normal operation of the vehicle.

## III. ANALYSIS OF FLIGHT TEST DATA OF A TOWED VEHICLE (DSRV)

Analysis of the flight test data of the Deep Submergence Research Vehicle (DSRV), supplied by the Naval Coastal Systems Laboratory, was done in the following three stages.

In the first phase, the digitized data of Runs 43 and 52 was analyzed for the following variables:

(a) Input - A short duration (7 sec) rudder pulse. It is to be noted that beyond the first 10 seconds the digitized data was clamped to a constant value.

(b) Output - Yaw variable. It was noted visually that the behavior of this variable in Run 43 was distinctly different than in Run 52. Specifically, the response in Run 52 was relatively sluggish suggesting that its towing conditions were different than for Run 43. That is, we feel that other than a different rudder input, there could be other factors causing fundamental differences between these runs. Among the suspected factors are different stern plane and/or bow plane inputs, and different bridles.

The errors in identification seemed reasonably low considering that the data were obtained via digitization of a strip chart recording. These errors are of the following magnitudes.

| Run | Order 2 | Order 4 | Order 6 |
|---|---|---|---|
| 43 | 8.4% | 3.9% | 3.0% |
| 52 | 6.8 | 4.4 | 3.1 |

Graphical display of reconstructed outputs can be seen in Figures 16 and 17. The identified transfer functions are shown on the figures.

In the second phase multiple-output analysis of the towed vehicle data was carried out (the approach in Appendix B for estimating the noise correlation matrix is used). The variables were

(a) Input - Short rudder pulse as described earlier

(b) Output - Yaw and roll variables. The yaw variable was discussed above. Similar remarks hold for the roll variable inasmuch as its modes in Runs 43 and 52 are distinctly different.

The errors in identification are reasonably low considering again that the data were obtained via digitization of a strip chart recording.

| Run | Order 2 (Yaw, Roll) | Order 4 (Yaw, Roll) |
|---|---|---|
| 43 | 15,8% | 9,4% |
| 52 | 7,6% | 6,4% |

Graphical display of the reconstructed outputs and the identified transfer functions, can be seen in Figures 18 and 19.

In the third phase analysis of a high frequency component, clearly visible in the Run 52 Roll, was carried out. Neither the fourth nor the sixth order identifications had detected this tremor. Two techniques were used:

Technique 1: Identification of the predominent second order mode was first carried out. Call this component in roll as $\phi_p(t)$. This was subtracted from the original roll data to yield a residual,

$$\phi_r(t)=\phi(t)-\phi_p(t)$$

The residual $\phi_r(t)$ was then used as the output data in a second order identification.

Technique 2: The data of the Run 52 Roll variable was first filtered by a low-pass third order Chebyshev filter, with a cut-off frequency of 0.24 hz. Using this filtered output a second order identification was performed. In order to reduce the effect of filter transient, the

first four seconds of the filtered data were discarded.

With both techniques, identification was carried out using two different inputs. First, the short rudder pulse presented by NCSL was used, since it could be presumed that the tremor was a natural mode excited by the rudder input. Then, a unit pulse located at $t = 0$ was used. In both cases, only 16 seconds of data were analyzed in order to reduce the effect of the long period of zero input.

Graphical displays of the results are found in Figures 20-22. The following observations arise:

Observation 1: Figure 20 compares the residual and filtered output for the entire run, and for the 16 seconds used for identification. Note that the residual behaves in such the same way as does the filtered output, both exhibiting a fairly regular pattern. The conclusion is that either technique can reveal the presence of a high frequency mode, and that a tremor characteristic of such a mode is present in the roll output.

Observation 2: The program does detect the tremor. With rudder input the poles were identified as $+0.12 \pm j\ 1.719$ with the filtered output, and as $+0.12 \pm j\ 1.274$ with the residual output (Figure 21). With a unit pulse input, the poles were identified as $+0.07 \pm j\ 1.743$ with the filtered output, and as $+0.13 \pm j\ 1.279$ with the residual output (Figure 22). Counting the number of peaks in the tremor gave an estimated frequency of about 1.8 rad/sec. The lower frequency of the identified system with the residual output is apparently due to the fact that in the identification of the filtered output 16 seconds of data were used starting at time $t = 4$ sec, whereas in the identification of the residual output 16 seconds of data starting at time $t = 0$ were used.

Observation 3: Identification of the filtered output has very satisfying aspects. Note the close relationship of the reconstruction with the true output, both in terms of frequency and phase. The initial phase shift with the rudder input is attributed to the rudder energy being concentrated at a time somewhat later than with the inpulse input. The fact that the technique gave unstable poles is because the tremor provides a continuing, non-decreasing output in the interval where the input is identically zero (or constant).

Observation 4: The fact that the unit pulse input gave substantially the same results as the rudder input implies that the high frequency tremor need not be a natural mode excited by the rudder and could well be due to wave-motion. Filtered rudder data (not shown) indicated the presence of a tremor of approximately the same frequency as was observed in the filtered roll data. It is possible that this high frequency rudder input continued for the entire 64 seconds of data, but was not recorded due to the limitations of the digitization technique used. This could have excited a natural high frequency roll mode. It is also possible that this rudder motion was caused by the same wave action that caused the roll tremor, and the rudder actually provided no further high frequency input.

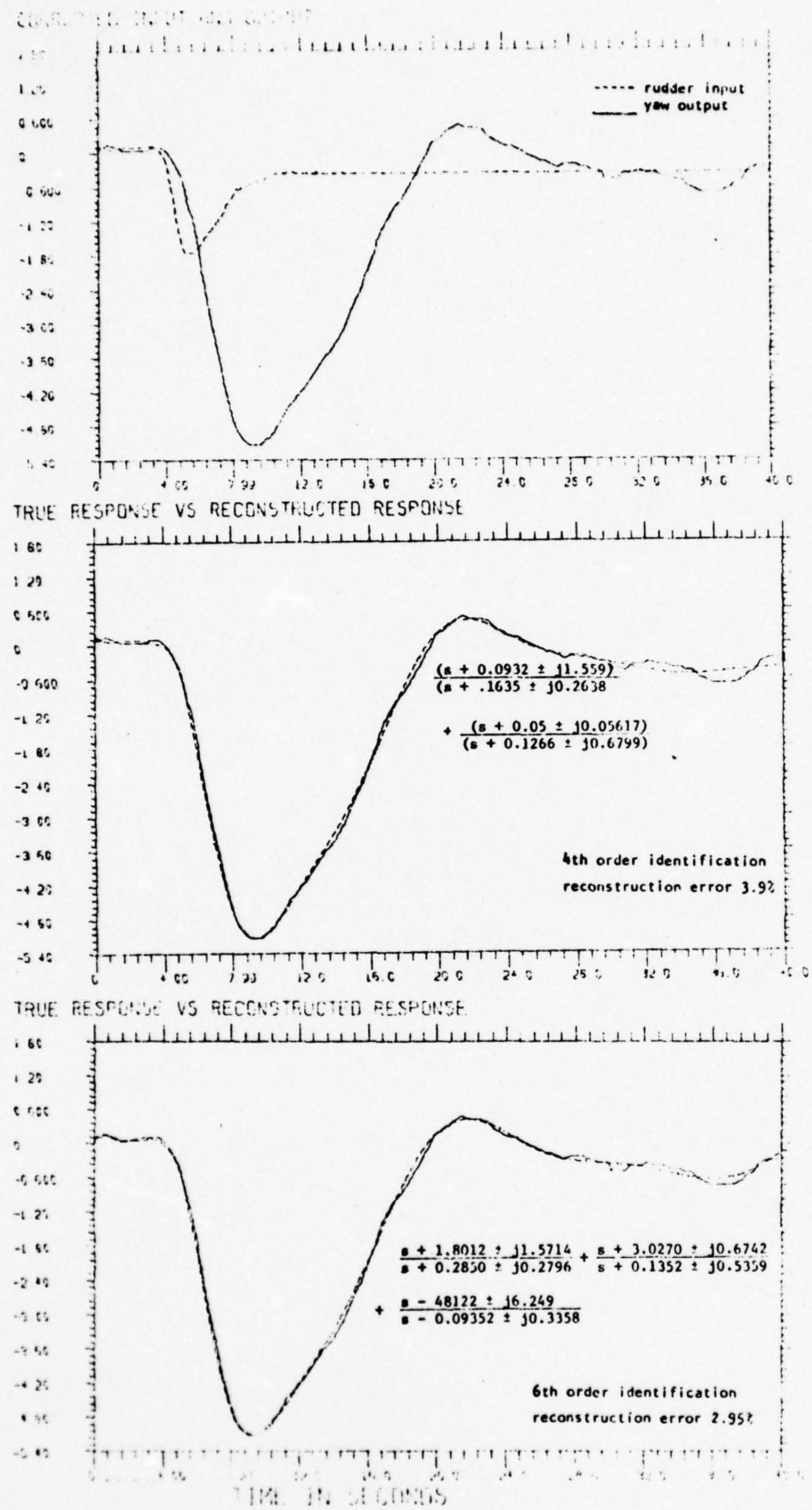
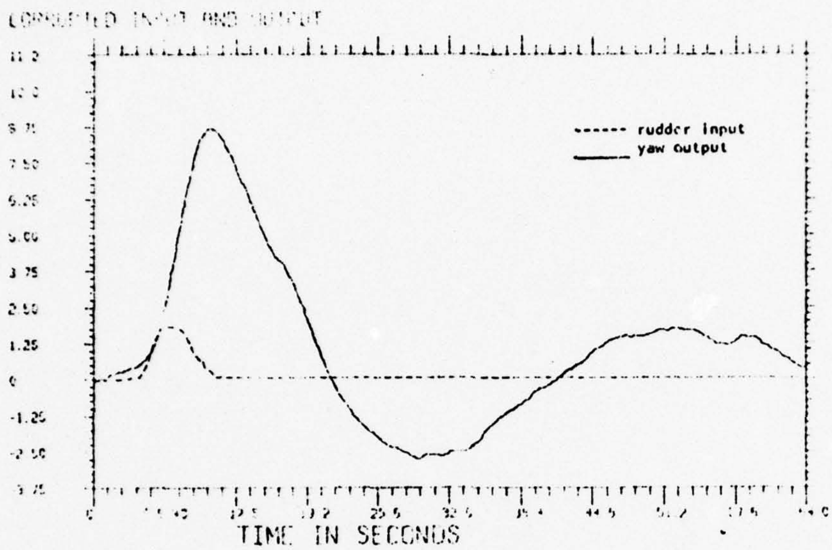
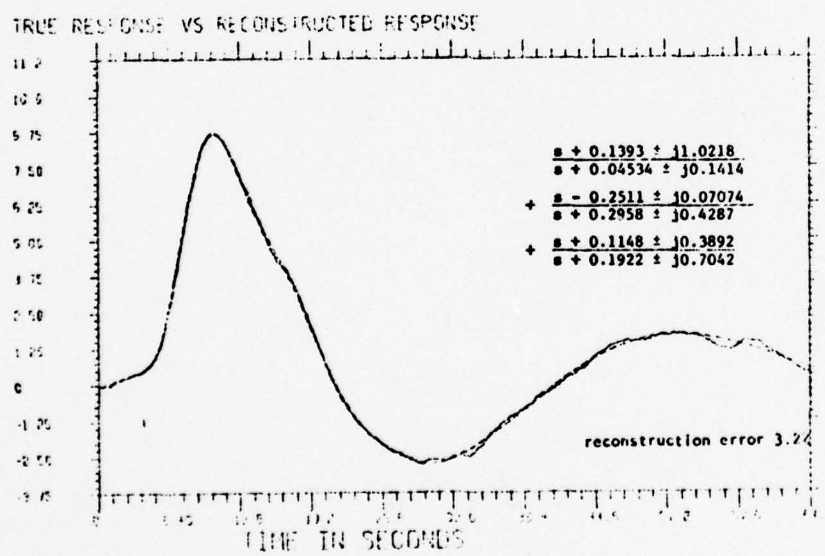



Fig. 16. Run 43 - Fourth and sixth order system identification from rudder and yaw data




$$\frac{s + 0.1393 \pm j1.0218}{s + 0.04534 \pm j0.1414} + \frac{s - 0.2511 \pm j0.07074}{s + 0.2958 \pm j0.4287} + \frac{s + 0.1148 \pm j0.3892}{s + 0.1922 \pm j0.7042}$$

Fig. 17. Run 52 - Sixth order systems identification from rudder and yaw data

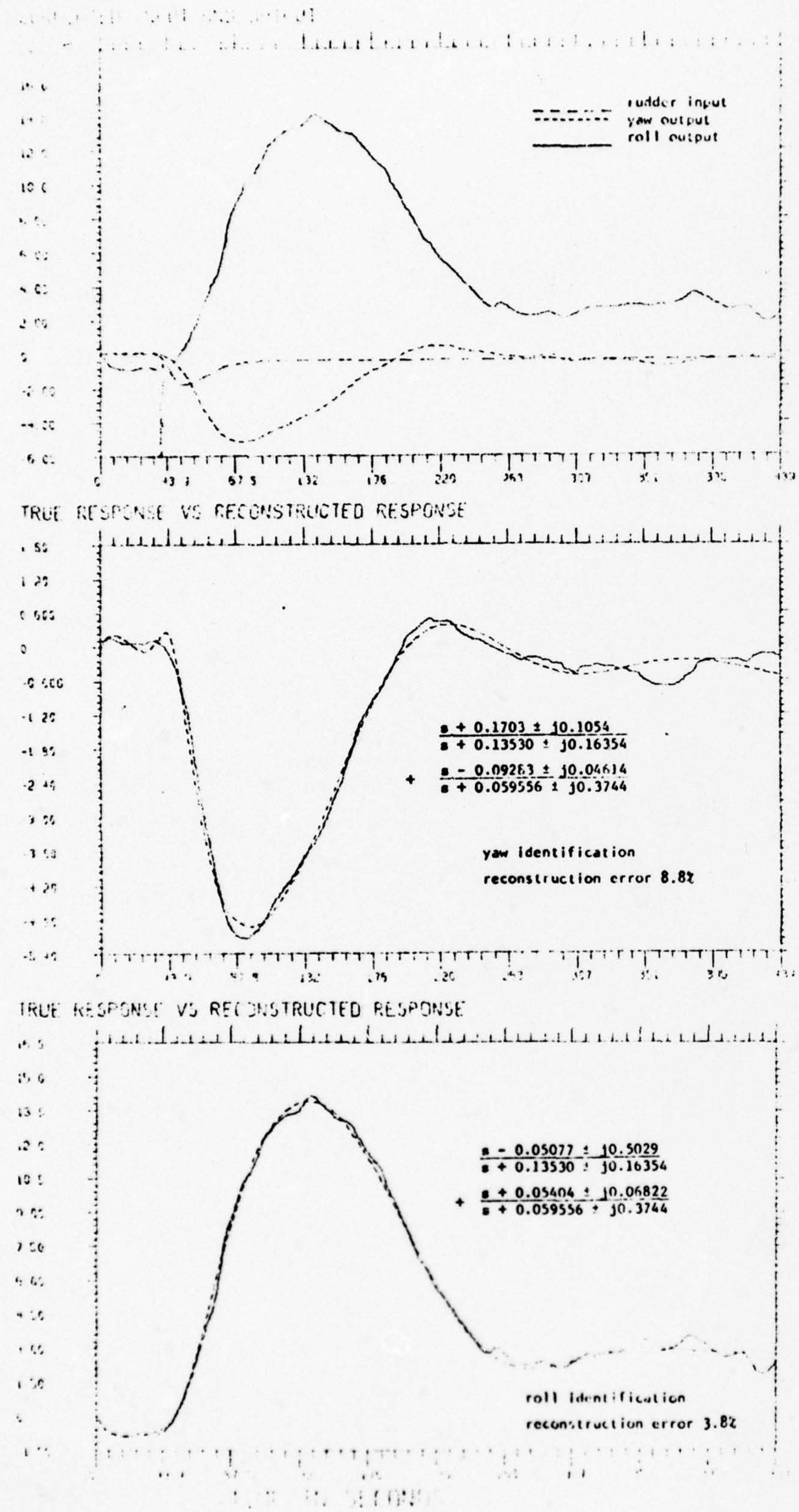



Fig. 18. Run 43 - Fourth order common mode identifications from yaw, roll and rudder data

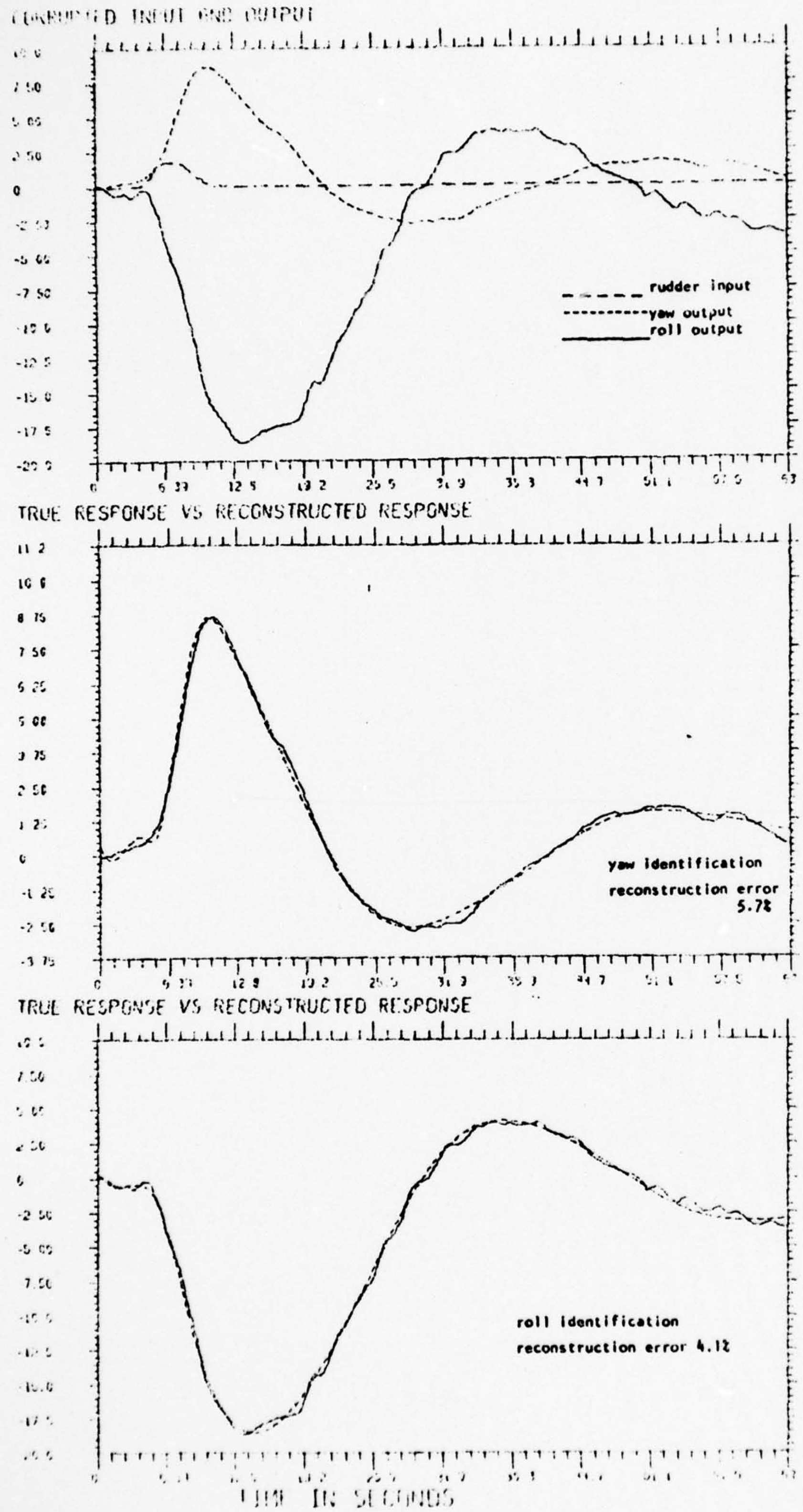
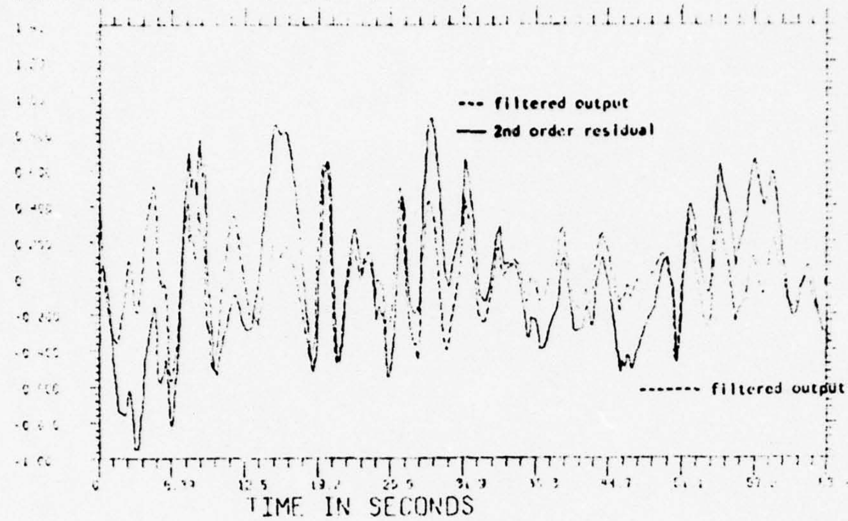



Fig. 19. Run 52 - Fourth order common mode identifications from yaw, roll and rudder data



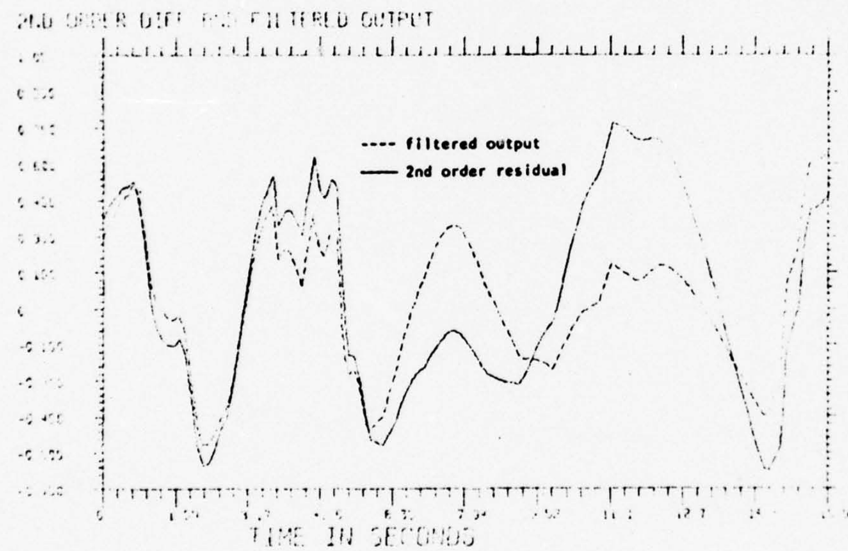




**Fig. 20.** Second order residual and filtered output from Run 52 Roll, showing tremor in data

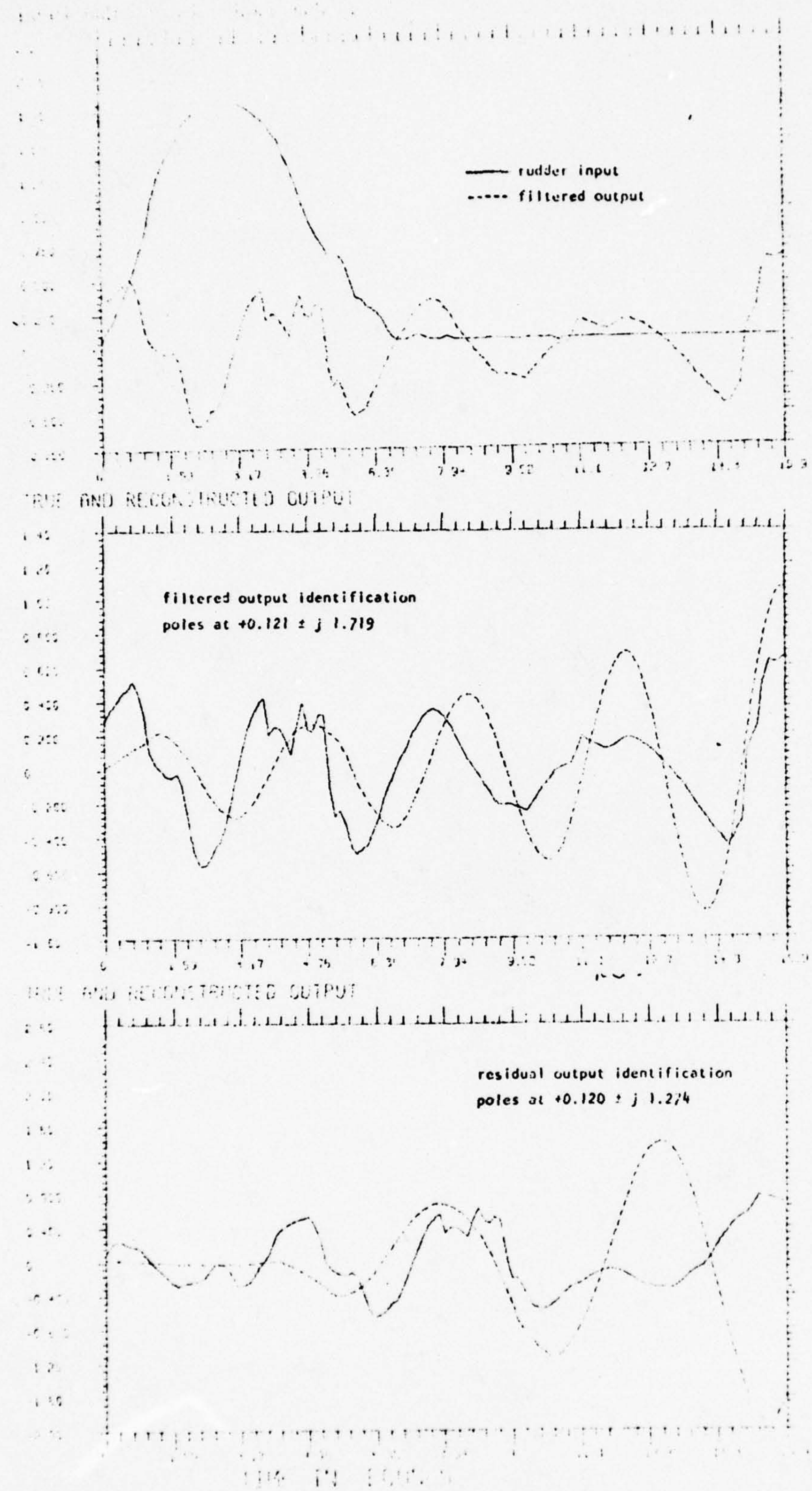
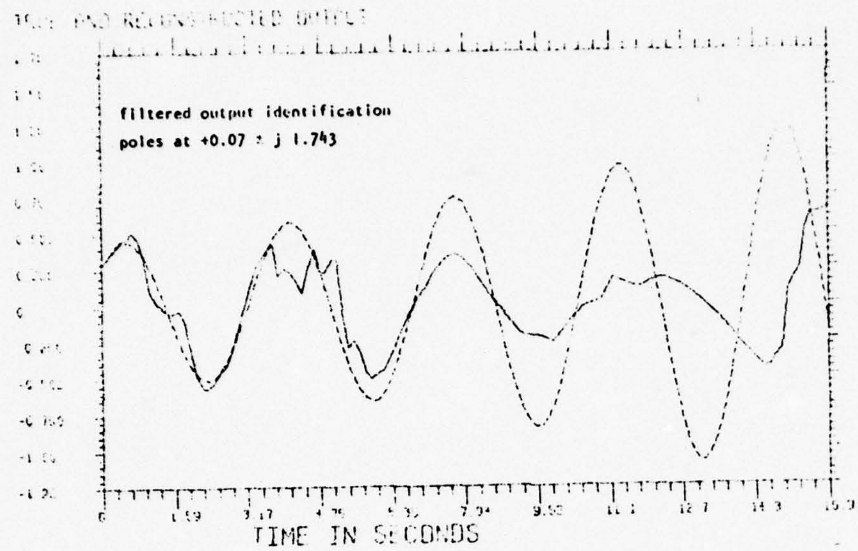



Fig. 21. Tremor identification with rudder input



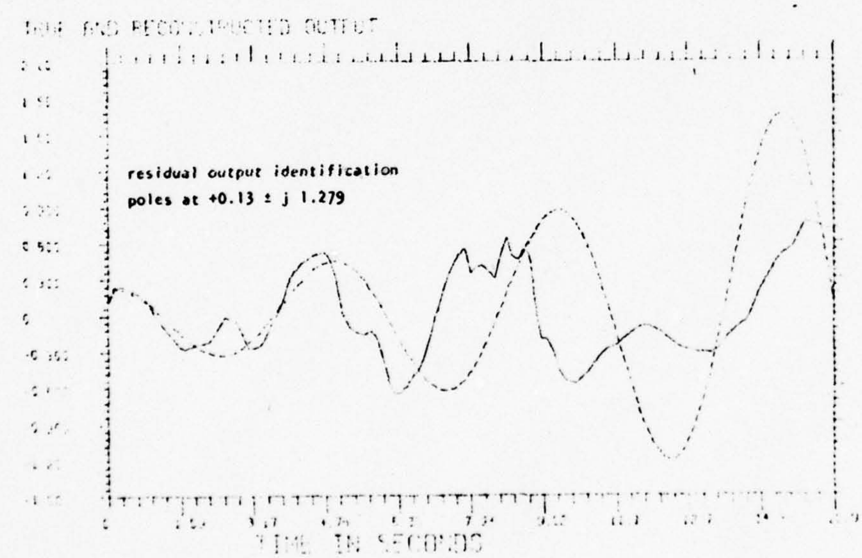




Fig. 22. Tremor identification with unit pulse input

# IV. UNSTABLE VEHICLE DYNAMICS IDENTIFICATION

It is not uncommon that the dynamics of the basic vehicle, an aircraft or a submerged craft, turn out to be unstable. Stable behavior must then be achieved through corrective feedback. Usually, the corrective feedback is provided electronically, but in the case of small diver-vehicles it may be provided manually. The situation is depicted in Fig. 23. The purpose of this chapter is to investigate the feasibility of applying the Gram technique to the identification of unstable vehicles. The single-input, single-output GRAM computer program was extended to include the feedback loop and the compensator in digital form. The program accepts a command input, determines the actual input to the control surface, and then carries out identification on the basis of this input and the vehicle response. Further, after identification, the feedback configuration is again used to determine the response of the identified vehicle to the original command input and, in turn, the reconstruction error. A two-man submersible is used here for the simulation studies performed.

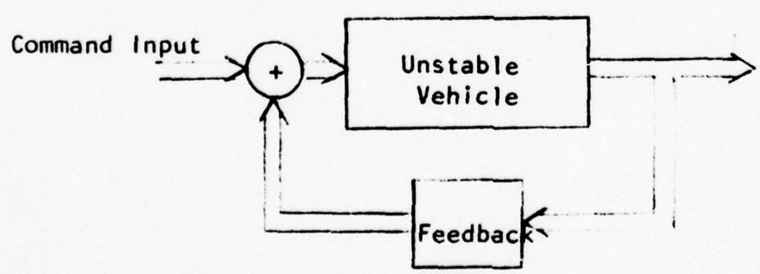



Fig. 23. Stabilization of Vehicle

## 4.1 TWO MAN SUBMERSIBLE

The transfer dynamics of two-man SUB, as predicted by the RGEORGE program and supplied to us by the Naval Coastal Systems Laboratory, is given below for ready reference. The bow planes were extended, the speed was 4.5

knots.

Longitudinal Dynamics

$$H(s) = \begin{bmatrix} u(s)/\delta_s(t) \\ w(s)/\delta_s(t) \\ \theta(s)/\delta_s(t) \end{bmatrix} \quad \begin{matrix} \text{ft./sec./degree} \\ \text{ft./sec./degree} \\ \text{dimensionless} \end{matrix}$$

$$= \frac{1}{D(s)} \begin{bmatrix} -0.003497s^3 & -0.0003947s^2 & +\ 0.0002482s & -0.000018053 \\ -0.04395s^3 & -0.03110s^2 & -0.005468s & -0.000067012 \\ & -0.30575s^2 & -0.174192s & -0.024776 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{(-0.003497)(s+0.35432)(s-0.11848)(s-0.12297)}{(s+0.34925)(s+0.27437)(s-0.06364)(s-0.2327)} \\[2ex] \dfrac{(-0.04395)(s+0.41995)(s+0.013232)}{(s+0.34925)(s-0.06374)(s-0.23227)} \\[2ex] \dfrac{(-0.30575)(s+0.29535}{(s+0.34925)(s-0.06364)(s-0.23227)} \end{bmatrix}$$

where the common denominator polynomial D(s) is

$$D(s) = s^4 + 0.327713s^3 - 0.073928s^2 - 0.019137s + 0.0014164$$

Single variable identification was performed on s(s) and $\theta(s)$, therefore their transfer functions are written as follows

$$\frac{w(s)}{\delta_s(s)} = \frac{1}{D_3(s)}(-0.04395s^2 - 0.019038s - 0.000244)$$

$$\frac{\theta(s)}{\delta_s'(s)} = \frac{1}{D_3(s)}(-0.30575 - 0.090303)$$

where

$$D_3(s) = s^3 + 0.053342s^2 - 0.088564s + 0.0051623$$

Lateral Dynamics

$$\frac{\dot{\psi}(s)}{\delta_r(s)} = \frac{1}{D(s)}[0.05286s^3 + 0.03810s^2 + 0.021775s + 0.006405]$$

$$= \frac{1}{D(s)}(0.05286)(s + 0.1486 \pm j\ 0.51374)(s + 0.42362)$$

$$\frac{\phi(s)}{\delta_r(s)} = \frac{1}{D(s)}\,[0.01572s^2 - 0.01796s - 0.020133]$$

$$= \frac{1}{D(s)}\,(0.01572)(s + 0.69637)(s - 1.8390)$$

where

$$D(s) = s^4 + 0.92823s^3 + 0.53444s^2 + 0.18314s + 0.023489$$

## 4.2 STUDY OF $w(s)/\delta_s(s)$

The feedback setup is shown in Fig. 24. The value of the gain used to stabilize the system was determined by a computer program. The closed loop poles for this gain, $K = -0.15$, turned out to be

$$|z| = 0.97289,\ 0.97618,\ 0.99057,\ 0.99057$$

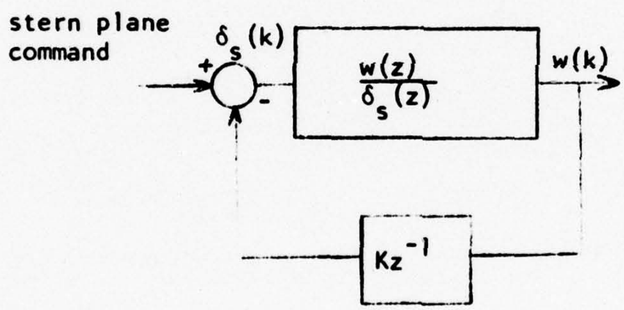



Fig. 24. Stabilization of w.

The results of identification for two different runs are given in Tables 4.1 and 4.2. In Run 1 the command input was a square wave with a period of 10 sec, followed by a slow exponential (exp (-0.2t)). In Run 2, the command input was a unit step. In either case the response variable data, w(k), was corrupted by 5% (rms) noise. It is clear from the results that the first input enables successful identification (see Fig. 25) while the step input

does not. Several other command inputs were also attempted for which the results are not given. In particular, a doublet with 10 sec. duration yielded successful identification.

## 4.3 STUDY OF $\theta(s)/\delta_s(s)$

The feedback setup is shown in Fig. 26. The compensator in the forward path was first designed by root locus method in the s-plane. This compensator turned out to be s/(s+2). The value of the gain (in the feedback path) was determined by a computer program. The closed loop poles for this gain, K = -3.5, turned out to be

$$|z| = 0.91449,\ 0.91449,\ 0.99079,\ 0.99079$$

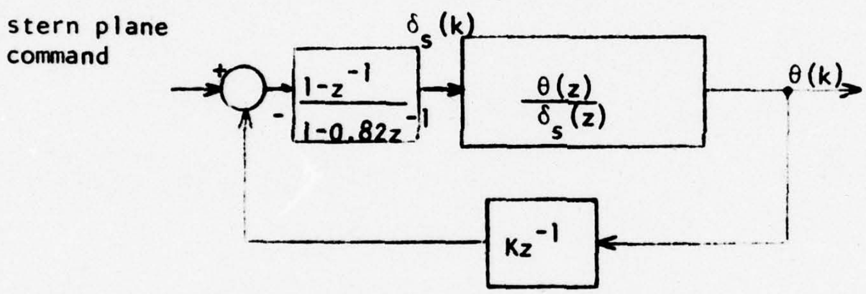



Fig. 26. Stabilization of pitch

The results of identification are presented only for one run, Run 3. In this run, the command input was a doublet of duration 10 sec. It is clear from Table 4.3 and Fig. 27 that the identification, as judged from the poles and the reconstruction, is successful. Other runs performed included a command input as in Run 1 (section 4.2) and a step command. The former resulted in excellent identification while the latter, as might be expected, did not.

## 4.4 Study of Lateral Dynamics

Although the lateral dynamics is stable, identification runs were performed on the roll and yaw rate variables. The results of identification are presented in Table 4.4 and 4.5. The rudder input in each of these runs, Run 4 and Run 5, was the same as in Run 1. That is, the rudder was deflected in a square-wave manner, with 10 sec. period, followed by an exponential restoration (exp (-0.2 t)). The results of reconstruction are shown in Figures 28 and 29.

Discussion - From the experiments performed it appears that the Gram technique can be used to identify the dynamics of an unstable vehicle. To insure the success of the procedure, the unstable vehicle must be embedded in a corrective feedback loop such that the feedback system is stable. The input and output of the vehicle itself are then used for the identification procedure. This requirement is, in fact, quite mild inasmuch as during the actual tests the vehicle is made stable through pilot feedback, or through automatic feedback. The next point to note is the following. After identification has been performed, the reconstructed output must be computed by use of the same feedback system as used before. This will be easily possible when the feedback is electronic and its transfer function description is available. However, when the feedback is manual, such description will be unavailable; therefore, it will not be possible to compute the reconstruction error on a feedback basis.

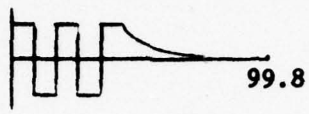
Reconstruction output computed on an open loop basis can be expected to diverge from the measured output, even when the vehicle parameters are identified reasonably well.

**TABLE 4.1**

**Results of Vehicle Dynamics Identification for a Two Man Submersible**

**Analysis of $w/\delta_s$**

**Command Input**




$\sigma_w = 0.0$ $\sigma_v = 0.05$

---

$$\frac{w(s)}{\delta_s(s)} = \frac{-0.043163\, s^2 - 0.020603\, s - 0.00024805}{s^3 + 0.0898481\, s^2 - 0.0984324\, s + 0.00578694}$$

Poles: -0.384763
0.0655826
0.229333

Reconstruction Error (RMS) 0.48% for Q =0.99

---

Other values of Q yielded larger errors: 1.966% for Q = 0.96

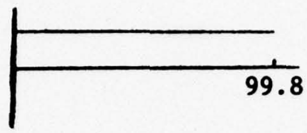
0.563% for Q = 0.98

TABLE 4.2

Results of Vehicle Dynamics Identification for a Two Man Submersible

Analysis of $w/\delta_s$

Command Input




$\sigma_w = 0.0$ $\sigma_v = 0.05$

---

$$\frac{w(s)}{\delta_s(s)} = \frac{-0.015291\ s^2 - 0.31351\ s\ 0.41086}{s^3 + 3.02462\ s^2 - 2.51836\ s - 4.15994}$$

Poles: -0.931123
1.31193
-3.40542

Reconstruction Error (RMS): 7713% for Q = 0.96

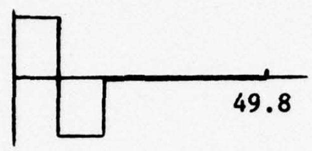
---

TABLE 4.3

Results of Vehicle Dynamics Identification for a Two Man Submersible

Analysis of $\theta/\delta_s$

Command Input




$\sigma_w = 0.0$ $\sigma_v = 0.05$

---

$$\frac{\theta(s)}{\delta_s(s)} = \frac{0.002777738\ s^2 - 0.302842\ s - 8.44054}{s^3 + 0.0326661\ s^2 - 0.0838420\ s + 0.00487453}$$

Poles : -0.330748
0.0625812
0.235501

Reconstruction Error(RMS): 0.754% for Q = 0.99

---

Other values of Q yielded larger errors: 0.780% for Q = 0.96

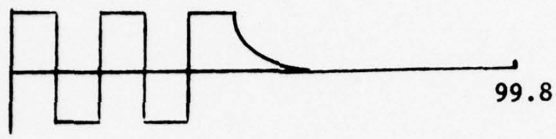
0.758% for Q = 0.98

TABLE 4.4

**Results of Vehicle Dynamics Identification for a Two Man Submersible**

**Analysis of $\phi/\delta_r$**

**Input**




$\sigma_w = 0.05$ $\sigma_v = 0.05$

---

$$\frac{\phi(s)}{\delta_s(s)} = \frac{0.0301947\ s^3 - 0.0324445\ s^2 + 0.145450\ s - 0.218783}{s^4 + 6.04558\ s^3 + 3.05609\ s^2 + 1.67827\ s + 0.246480}$$

Poles: $-0.154865 \pm 0.460733\ j$
$-0.188050$
$-5.54780$

Reconstruction Error (RMS): 2.15% for Q = 0.96

---

Other values of Q Yielded larger errors: 2.970% for Q = 0.98

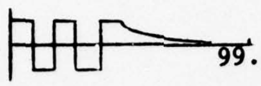
5.483% for Q = 0.99

TABLE 4.5

Results of Vehicle Dynamics Identification for a Two Man Submersible

Analysis of $\dot{\psi}/\delta_r$

$\sigma_w = 0.05$




$\sigma_v = 0.05$

---

$$\frac{\dot{\psi}(s)}{\delta_s(s)} = \frac{-0.0519855\ s^3 - 0.0375124\ s^2 - 0.0198798\ s - 0.00598798}{s^4 + 0.924383\ s^3 + 0.495825\ s^2 + 0.173456\ s + 0.0213749}$$

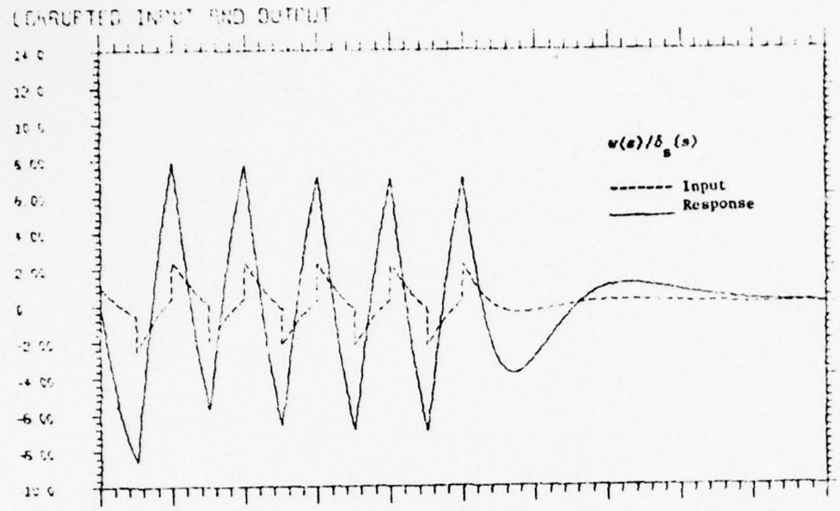
Poles: $-0.133581 \pm 0.455095\ j$
$-0.213805$
$-0.444416$

Reconstruction Error (RMS): 1.49% for Q = 0.96

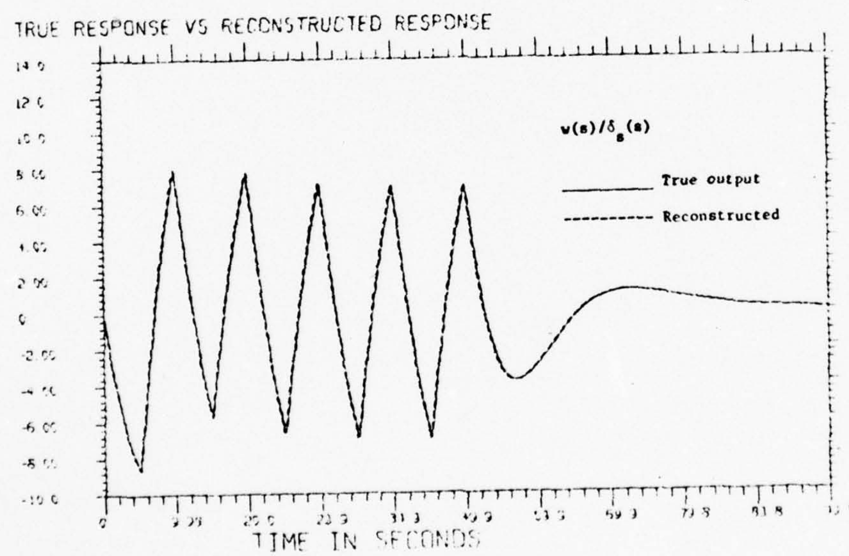
---

Other values of Q yielded larger errors: 3.97% for Q = 0.98

5.07% for Q = 0.99

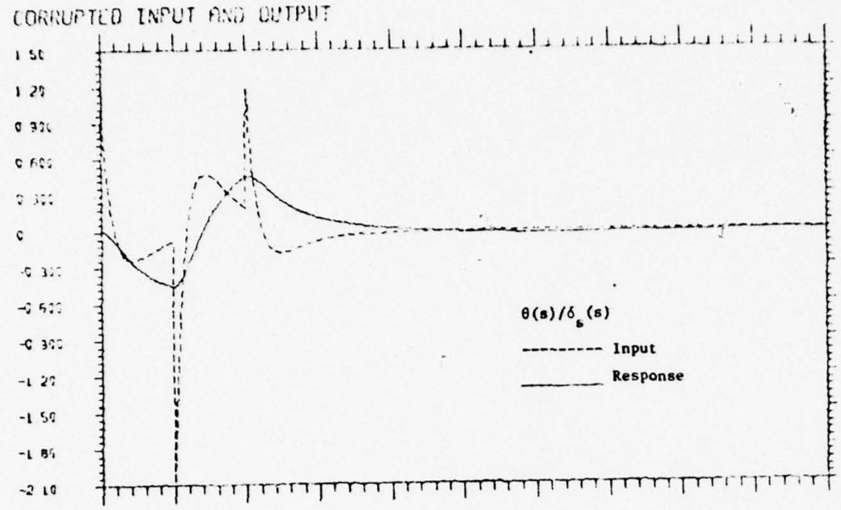



(a) Response vs stern plane

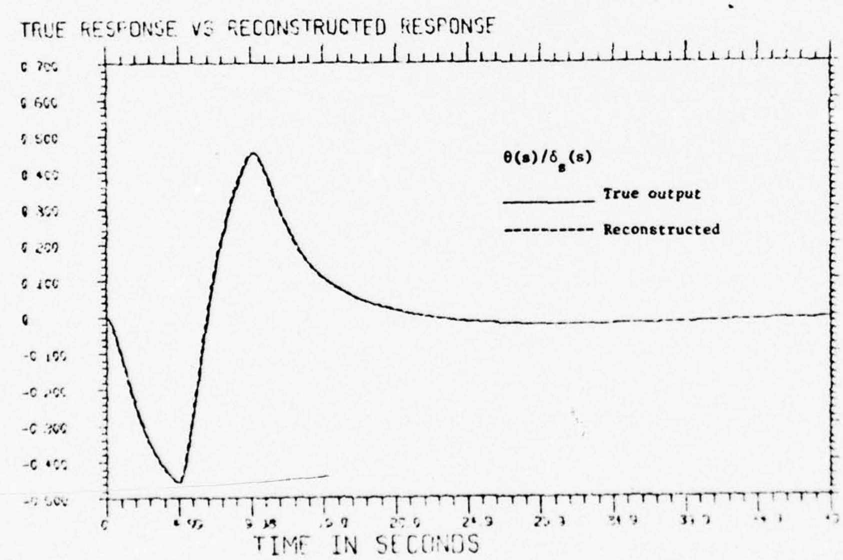



(b) Reconstructed output

Fig. 25. Identification of an unstable vehicle: $\sigma_w = 0$, $\sigma_q = 0.05$, $\Delta = 0.1$, $Q = 0.99$

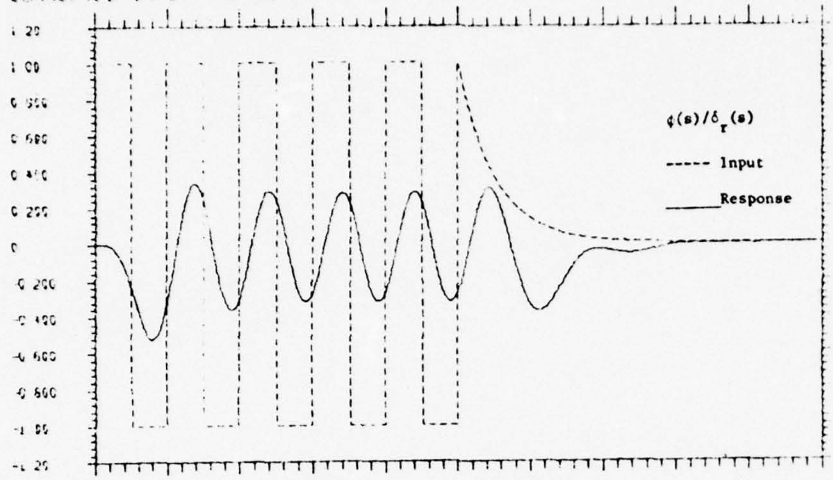



(a) Response vs stern plane

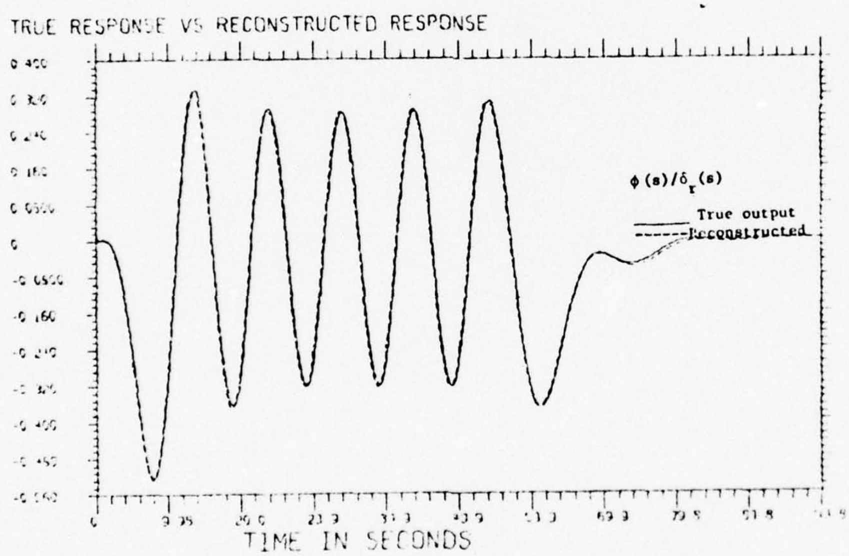



(b) Reconstructed output

Fig. 27. Identification of an unstable vehicle: $\sigma_w = 0$, $\sigma_q = 0.05$, $\Delta = 0.1$, $Q = 0.98$

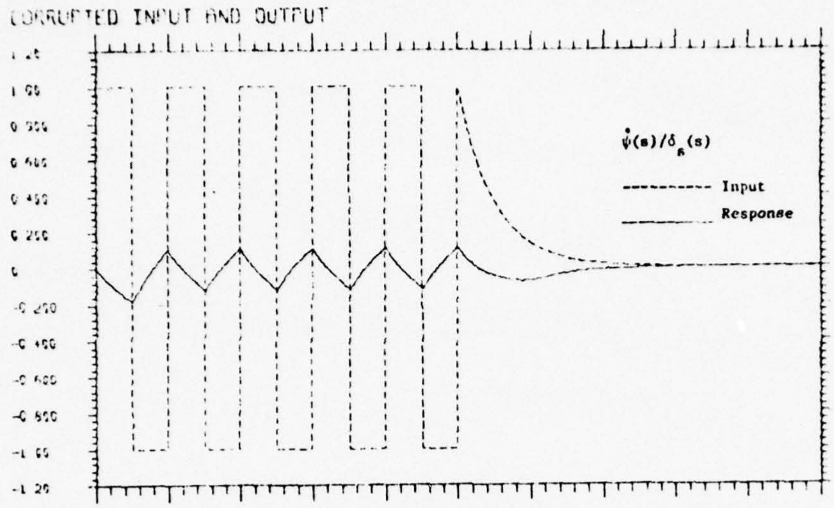



(a) Response vs rudder

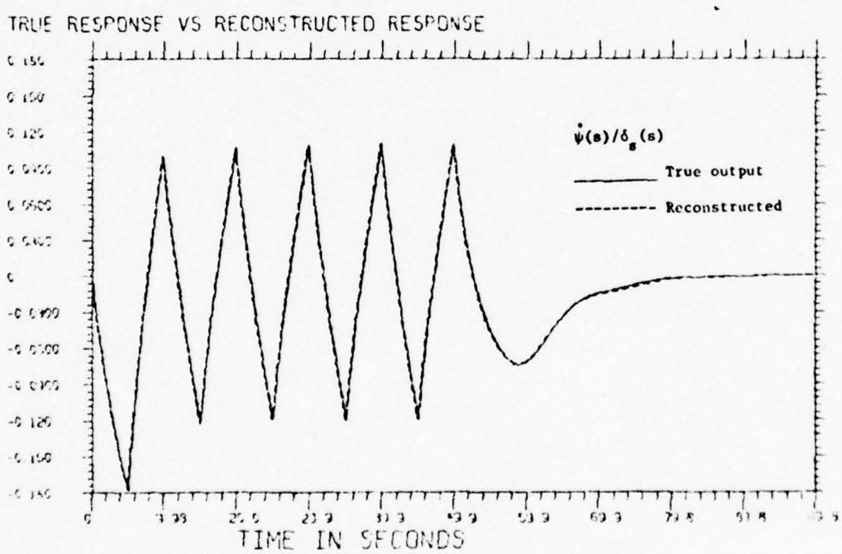



(b) Reconstructed output

Fig. 28. Identification of an unstable vehicle:
$\sigma_w = 0.05$, $\sigma_q = 0.05$, $\Delta = 0.1$, $Q = 0.96$




(a) Response vs rudder




(b) Reconstructed output

Fig. 29. Identification of an unstable vehicle:
$\sigma_w = 0.05$, $\sigma_q = 0.05$, $\Delta = 0.1$, $Q = 0.96$

## V. A TWO STAGE IDENTIFICATION SCHEME: GRAM-TAYLOR

As stated in Chapter I, the Gram technique does not require any initial estimates of the vehicle parameters; additionally, it is noniterative. The latter, while being an important advantage, points to the fact that the method lacks the ability to improve results obtained through a single pass. On the other hand, the NASA program developed by Taylor and Iliff [3] suffers from somewhat complementary defects. Specifically, its iterative algorithm does not always converge (to the true values) when the initial estimates are not reasonably close to true values. It would therefore appear that cascading the Gram and Taylor techniques could alleviate the short comings of either method.

The cascading of MGRAM to TAYLOR (we shall refer to the NASA program as TAYLOR) involved three things: 1) conversion of the transfer function matrix obtained from MGRAM to a state model, 2) implementation of the TAYLOR program on our computer and 3) testing of GRAM-TAYLOR. We shall discuss each of these three in brief.

### 5.1 STATE MODEL

Having identified the transfer matrix $H(s)$ of the vehicle by use of MGRAM, one can obtain the state model by the procedure given in Chapter II (Section 2.5). An alternative, and somewhat simpler, procedure is available for the case where the input is a single variable. Specifically consider that

$$y(s) = H(s)\ u(s) \qquad (1)$$

where $y$ is m-dimensional, us is scalar and $H$ is the m-dimensional column vector

$$H(s) = \frac{1}{D(s)} \begin{bmatrix} f_{10} + f_{11}s + \cdots + f_{1,N-1}s^{N-1} \\ \vdots \\ f_{m0} + f_{m1}s + \cdots + f_{m,N-1}s^{N-1} \end{bmatrix} \quad (2)$$

$$D(s) = s^N + a_{N-1}s^{N-1} + \cdots + a_1 s + a_0$$

Then it can be shown that a possible slate model for (1) is the following

$$\dot{x} = Ax + bu \quad (3)$$

$$y = Fx$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & . & . & 0 \\ 0 & 0 & 1 & . & . & 0 \\ & & & . & . & \\ 0 & 0 & 0 & . & . & 1 \\ -a_0 & -a_1 & -a_2 & . & . & -a_{N-1} \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ . \\ 0 \\ 1 \end{bmatrix} \quad (4)$$

$$F = \begin{bmatrix} f_{10} & f_{11} & \cdots & f_{1,N-1} \\ f_{20} & f_{21} & \cdots & f_{2,N-1} \\ & & \vdots & \\ f_{m0} & f_{m1} & \cdots & f_{m,N-1} \end{bmatrix} \quad (5)$$

## 5.2 TAYLOR PROGRAM

The TAYLOR program was adapted for use on our computer. Certain modifications were made on the copy of the program supplied to us to correctly implement the iterative procedure. Appendix A provides a discussion of the modifications made. It was tested on the simulated dynamics of a jet-aircraft whose parameters were taken from reference [8]. The following observations:

(a) if the measured outputs included <u>all of the state</u> variables, the program converged and found all of the unknown parameters even when the measurements were corrupted with noise (5% rms). The initial estimates were all taken to be zero.

(b) When the measured output consisted of only one of the state variables, specifically the pitch angle, the algorithm failed to converge.

### 5.3 GRAM-TAYLOR TESTS

The longitudinal dynamics of a Six man submersible, considered earlier in chapter 2, was studied. The initial conditions and the stern plane input were the same as in Run 5 of chapter II, namely

$$\delta_s(t) = \begin{cases} 0.5^o & \text{for} \quad 0 \leq t \leq 62.5 \text{ sec} \\ -0.5^o & \text{for } 62.5 < t \leq 125.0 \text{ sec} \\ 0 & \text{for } 125 < t \leq 250 \text{ sec} \end{cases}$$

$$u(0) = w(0) = \theta(0) = \dot{\theta}(0) = 0$$

The output measurements u, w and $\theta$ were corrupted by 5% rms noise. Identification via MGRAM yielded the same results as in Table 2.2 repeated in Table 5.1 for convenience. The state model was developed in the form discussed on the previous page specifically, the matrices of equation (2) turned out to be

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.0011633 & -0.023646 & -0.13347 & -1.7096 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$F = \begin{bmatrix} -0.0000425 & +0.0000396 & -0.00290 & -0.00221 \\ -0.0000533 & -0.00291 & -0.0366 & -0.0100 \\ -0.00982 & -0.200 & -0.377 & +0.00783 \end{bmatrix}$$

The vector of parameters that the TAYLOR program was given to improve upon was a 20 - dimensional vector. It consisted of the four bottom entries of A, and all of the twelve entries of the matrix F. Since none of the state variables is measured directly, the weighting matrix for the output error in the algorithm [3] is taken to be

$$D1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2173 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 82 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.018 \end{bmatrix}$$

The result of iterative improvement by the TAYLOR program are presented in Table 5.2.

A second experiment was performed on the simulated data of the six man submersible. In this experiment the nonzero initial conditions and the stern plane input were chosen to be the same as in Run 6 of chapter II, namely

$$\delta_s(t) = \begin{cases} 0.5^o & \text{for} \quad 0 \leq t \leq 62.5 \text{ sec} \\ -0.5^o & \text{for} \quad 62.5 < t \leq 125 \text{ sec} \\ 0 & \text{for} \quad 125 < t \leq 250 \text{ sec} \end{cases}$$

and

$$u(0) = -0.018$$

$$w(0) = -0.022$$

$$\theta(0) = -4.31$$

$$\dot{\theta}(0) = 0.0$$

Identification via MGRAM yielded the same results as in Table 2.3 of chapter II, repeated here in table 5.3 for convenience. The state model was developed as in the first experiment, specifically,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.0014923 & -0.030319 & -0.16796 & -2.1867 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$F = \begin{bmatrix} -0.0000547 & +0.0000535 & -0.00376 & -0.00271 \\ -0.0000682 & -0.00371 & -0.0467 & -0.00408 \\ -0.0126 & -0.257 & -0.453 & +0.0980 \end{bmatrix}$$

As in the first experment, a 20 - dimensional vector of initial estimates was formed and used in the TAYLOR program. Of course the stern plane input and the measured outputs (u, w and $\theta$; each corrupted by 5% rms noise) was also supplied to the program. The results of iterative improvement are shown in Table 5.4.

Discussion: In each of the two experiments above, the improvement of the apriori parameter estimates (obtained from MGRAM) was not appreciable although convergence of the Newton Method did occur. Lack of improvement in the identification of the micro-model is, as in the case of MGRAM,

largely due to the fact that the slow sampling rate and 125 sec doublet stern plane are not able to make this mode observable. Recall this mode is present only in the variable u. Typical computer run times on the IBM 360 were 250 sec for MGRAM, and 800 sec. for TAYLOR.

TABLE 5.1

Results of Longitudinal Dynamics Identification of a Six Man Submersible

Non-zero Initial conditions: $u(0) = 0.0$, $w(0) = 0.0$, $\theta(0) = 0.0$, $\dot{\theta} = 0.0$

Stern plane Input $\delta_s(t)$ = doublet, duration = 125 sec., amplitude = 0.5 degree

Ratio of Noise to Signal rms Values = 5% for each output variable.

Q Parameters: 0.933594, 0.953124, 0.970703, 0.983643

---

Identified transfer functions

z-domain

$$h(z) = \frac{1}{D(z)} \begin{bmatrix} -0.00125z^{-1} & +0.00248z^{-2} & -0.00122z^{-3} & -0.0000231z^{-4} \\ -0.000585z^{-1} & -0.0140z^{-2} & +0.0291z^{-3} & -0.0145z^{-4} \\ 0.0560z^{-1} & -0.330z^{-2} & +0.489z^{-3} & -0.175z^{-4} \end{bmatrix}$$

$$D(z) = 1 -2.51950z^{-1} +1.61875z^{-2} +0.326244z^{-3} -0.425364z^{-4}$$

s-domain

$$h(s) = \frac{1}{D(s)} \begin{bmatrix} -0.00221s^{3} & -0.00290s^{2} & +0.0000396s & -0.0000425 \\ -0.0100s^{3} & -0.0366s^{2} & -0.00291s & -0.0000533 \\ 0.00783s^{3} & -0.377s^{2} & -0.200s & -0.00982 \end{bmatrix}$$

$$D(s) = s^4 + 1.70962s^3 +0.133473s^2 +0.0236459s +0.00116332$$

$$= (s +0.00917945 \pm 0.113694\,j)\ (s + 0.0546324)\ (s + 1.63663)$$

Reconstruction Errors: 0.86% for u

1.06% for w

0.37% for $\theta$

TABLE 5.2

Results of Logitudinal Dynamic Identification of Six Man Submersible via TAYLOR program using estimates of MGRAM for initialization

Zero Initial conditions: $u(0) = 0$, $w(0) = 0$, $\theta(0) = 0$, $\dot{\theta}(0) = 0$
Stern Plane Input: $\delta_s(t)$ = doublet, duration = 125 sec, amplitude = 0.5°
Ratio of Noise to Signal rms Values = 5% for each response variable.
Convergency reached in 10 iterations.

---

Identified State Variable Matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.0011659 & -0.023491 & -0.13365 & -1.6961 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.0000425 & +0.0000416 & -0.00291 & -0.00215 \\ -0.0000533 & -0.00289 & -0.0362 & -0.00830 \\ -0.0098385 & -0.19904 & -0.370 & +0.00141 \end{bmatrix}$$

Characteristic Polynomial:

$$D(s) = s^4 + 1.7s^3 + 0.134s^3 + 0.0235s^2 + 0.00117$$

$$= (s+0.055218)(s+1.6224)(s+0.0092554 \pm j0.11370)$$

Error:

$$\varepsilon^2 = \sum_{k=1}^{K} \left|\left| \text{observed } y(k) - \text{reconstructed } y(k) \right|\right|^2_{D1}$$

Apriori $\varepsilon^2 = 0.00744$

10th Iteration $\varepsilon^2 = 0.00739$

Reconstruction Error: (Reconstructed y(k) vs observed y(k))

4.92 % for u

4.89 % for w

5.08 % for $\theta$

TABLE 5.3

Results of Longitudinal Dynamics Identification of a Six Man Submersible

Non-zero Initial conditions: $u(0) = -.018$, $w(0) = -.022$, $\theta(0) = -4.31$, $\dot{\theta} = 0.0$ Stern plane Input $\delta_s(t)$ = Doublet, duration=125sec, amplitude=0.5 degree

Ratio of Noise to Signal rms Values = 5% for each output variable.
Q Parameters: 0.931641, 0.953125, 0.970703, .984131

---

Identified transfer functions

<u>z-domain</u> $h(z) =$

$$\frac{1}{D(z)}\left[\begin{array}{lll|lll}
(-0.00146z^{-1} & +0.00321z^{-2} & -0.00202z^{-3} & (-0.0175 & +0.0283z^{-1} & -0.00457z^{-2} \\
 & & +0.000275z^{-4}) & & & -0.00633z^{-3} \\
(0.000371z^{-1} & -0.0159z^{-2} & +0.0300z^{-3} & (-0.0179 & +0.0179z^{-1} & -0.0243\ z^{-2} \\
 & & -0.0145z^{-4}) & & & -0.0240\ s^{-3}) \\
(0.0719z^{-1} & -0.369z^{-2} & +0.482z^{-3} & (-4.11 & +6.34z^{-1} & -0.520z^{-2} \\
 & & -0.186z^{-4}) & & & -1.71z^{-3}
\end{array}\right]$$

$$D(z) = 1 - 2.61323z^{-1} + 1.89615z^{-2} + 0.0522966z^{-3} - 0.335096z^{-4}$$

<u>s-domain</u> $h(s) =$

$$\frac{1}{D(s)}\left[\begin{array}{lll|lll}
(-0.00271s^{3} & -.00376s^{2} & +0.0000535s & (-0.0350s^{3} & -0.0758s^{2} & -0.00238s \\
 & & -0.0000547) & & & -0.00115) \\
(-0.00408s^{3} & -0.0467s^{2} & -0.00371s & (-0.0348s^{3} & -0.108s^{2} & +0.0402s \\
 & & -0.0000682) & & & +0.00238) \\
(\ 0.0980s^{3} & -0.453s^{2} & -0.257s & (-8.21s^{3} & -18.8s^{2} & -0.986 \\
 & & -0.0126) & & & +0.00929)
\end{array}\right]$$

$$D(s) = s^4 + 2.18668s^3 + 0.167964s^2 + 0.0303190s + 0.00149231$$
$$= (s + 0.00926847 \pm 0.113652\,j)(s + 0.0542944)(s + 2.11384)$$

Reconstruction Errors: 0.98% for u

1.31% for w

0.51% for $\theta$

TABLE 5.4

Results of Longitudinal Identification of Six Man Submersible via TAYLOR program using estimates of MGRAM for initialization.

Non Zero Initial Conditions: $u(0)=0.018$, $w(0)=0.022$, $\theta(0)=-4.31$, $\dot{\theta}(0)=0$
Stern Plane Input: $\delta_s(t)$ = doublet, duration = 125 sec, amplitude = 0.5°
Ratio of Noise to Signal rms Values = 5% for each response variable.
Convergence reached in 6 iterations.

---

Identified State Variable Matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.0015010 & -0.030043 & -0.16788 & -2.1623 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.0000548 & +0.0000614 & -0.00376 & -0.00229 \\ -0.0000684 & -0.00372 & -0.0460 & -0.00495 \\ -0.0127 & -0.255 & -0.44811 & -0.00125 \end{bmatrix}$$

Characteristic Polynomial:

$$D(s) = s^4 + 2.16s^3 + 0.168s^2 + 0.030s + 0.0015$$
$$= (s+0.055191)(s+2.0887)(s+0.0092310 \pm j0.11373)$$

Error: $$\varepsilon^2 = \sum_{k=1}^{K} ||\text{observed } y(k) - \text{reconstructed } y(k)||^2_{D1}$$

Apriori $\varepsilon^2 = 0.164$

6th Iteration $\varepsilon^2 = 0.00739$

Reconstruction Error: (reconstructed y(k) vs observed y(k))

4.92% for u

4.89% for w

5.08% for $\theta$

## VI. LINEAR MODEL FOR A MILDLY NONLINEAR SYSTEM

Linear identification of the dynamics of a six man submersible is studied while the maneuvers include the contribution of nonlinear terms. Specifically the purpose of this chapter is to study the degradation in identification of the linear model when the input-output data actually pertains to a maneuver in which the nonlinear force, or moment, terms make a non-negligible contribution. It is assumed, however, that the linear terms are dominant. To simulate the vehicle maneuvers required for this study the Navy program DYNAMIC, supplied to us by the Naval Coastal Systems Laboratory, was implemented on the IBM 360/65. This computer program predicts underwater vehicle responses by solving the equations of motion [9]. Two sets of stability derivatives pertaining to a six man submersible were supplied to us (see Appendix C):

<u>Set A</u> Linear Coefficients only

<u>Set B</u> Linear and important nonlinear terms. The latter were $X_{wq}$, $Z_{ww}$, $Z_{w|w|}$, $M_{w|w|}$, $Y_{v|v|}$ and $N_{v|v|}$

The maneuver chosen was a depth change maneuver starting from a steady state level flight at 200 feet, and descending to 300 feet via stern plane actuation only. The stern plane deflection was controlled automatically using pitch, pitch rate, depth, and depth rate feedback. This capability in DYNAMIC, allowed u, w, and $\theta$ to be maintained within tolerable limits so that the differences between the responses using Set B of coefficients, containing the nonlinear stability derivatives, and Set A, containing only linear terms, were mild. The values of the pertinent parameters used by DYNAMIC to execute the maneuver are given here.

| | | |
|---|---|---|
| IAUTO = 2 | ZORD = 300.0 | THORD = 0.0 |
| DSMAX = 5.0 | ZSEN = -0.02 | THSEN = -0.15 |
| DSDOT = 66.7 | ZDSEN = 0.05 | THDSEN = 0.6 |

The single input, three output representation of the linear system to be identified by MGRAM is described in the transfer function equation,

$$\begin{bmatrix} u(s) \\ w(s) \\ \theta(s) \end{bmatrix} = \begin{bmatrix} h_1(s) \\ h_2(s) \\ h_3(s) \end{bmatrix} \delta_s(s)$$

A sampling interval of 1.0 second is used and the total number of samples in each signal is 181. Two experiments were performed where the first assumed a system order equal to 4 and the second a system order equal to 6; all else remained the same between the two experiments. In each of the two experiments, two runs were made: one corresponding to the response based on the stability derivative Set A and the other based on Set B. Thus four runs were made as follows:

Run 1: Stability derivative set A, order of identification 4. The results of identification are shown in Table 6.1. The stern plane input for the desired maneuver is shown in Fig. 30. The vehicle response and the reconstructed output are shown in Fig. 31.

Run 2: Stability derivative set B, order of identification 4. The results of identification are shown in Table 6.1. The stern plane input for the desired maneuver is shown in Fig. 32. The vehicle response and the reconstructed output are shown in Fig. 33.

Run 3: Stability derivative set A, order of identification 6. The results of identification are shown in Table 6.3. The stern plane input for the desired maneuver was the same as in Run 1, Fig. 30. The vehicle response and the reconstructed output are shown in Fig. 34.

Run 4: Stability derivative set B, order of identification 6. The results of identification are shown in Table 6.3. The stern plane input for the desired maneuver was the same as in Run 1, Fig. 32. The vehicle response and the reconstructed output are shown in Fig. 35.

Discussion: A unit pulse response of the vehicle, obtained from program DYNAMIC and using the coefficients of set A, had earlier indicated an unstable longitudinal response (in the pitch variable). Each of the four runs confirmed this fact in that MGRAM identified this unstable mode. However, the reconstruction errors of the 4th order models, in Runs 1 and 2, were undesirable (see Fig. 31 and Fig. 33). Inspection of the responses indicates the presence of two natural frequencies; one appearing in both w and $\theta$ and the second in u (Fig. 31). Hence two complex pole pairs are needed. In addition, two decaying real exponentials are observed in the responses u and $\theta$, one in u and the other in $\theta$. These visual observations indicated that a 6th order model should be attempted. The improvement in the 6th order model over the 4th model is evident in Fig. 33. Futhermore, the closeness of the linear identified model for the vehicle with coefficients from set A and the linear model identified for the vehicle with coefficients from set B indicates that linear identification of dominantly linear system containing nonlinear contributions is indeed viable and a practical means of characterizing the system (compare Tables 6.3 and 6.4).

TABLE 6.1

Results of Longitudinal Dynamic Identification of a Six Man Submersive by MGRAM - u, w, and θ responses were generated by program DYNAMIC using Linear Coefficients only.

Identification Order N = 4

Depth Change Maneuver from 200 to 300 feet.

---

Identified transfer functions

$$h(s) = \frac{1}{D(s)} \begin{bmatrix} -0.00110s^3 & -0.000113s^2 & -0.0000251s & -0.00000142 \\ -0.0266s^3 & -0.00170s^2 & -0.000261s & -0.00000362 \\ -0.268s^3 & -0.153s^2 & -0.0105s & -0.00126 \end{bmatrix}$$

$$\begin{aligned} D(s) &= s^4 + 0.025793s^3 + 0.020030s^2 + 0.00049584s + 0.00012108 \\ &= (s + 0.031575 \pm j\ 0.090207)(s - 0.018679 \pm j\ 0.11361) \end{aligned}$$

Reconstruction Error: 34.7% for u

8.5% for w

3.3% for θ

TABLE 6.2

Results of Longitudinal Dynamic Identification of a Six Man Submersive by MGRAM - u,w, and $\theta$ responses were generated by program DYNAMIC using Linear and Nonlinear Coefficents.

Identification Order N = 4

Depth Change Maneuver from 200 to 300 feet.

---

Identification transfer functions

$$h(s) = \frac{1}{D(s)} \begin{bmatrix} -0.00143s^3 & -0.000246s^2 & -0.0000223s & -0.00000355 \\ -0.0271s^3 & -0.00120s^2 & -0.000316s & -0.00000619 \\ -0.176s^3 & -0.104s^2 & -0.00420s & -0.00113 \end{bmatrix}$$

$$D(s) = s^4 - 0.0021104s^3 + 0.019597s^2 - 0.000074644s + 0.00010438$$

$$= (s + 0.014577 \pm j\ 0.10449)(s - 0.015632 \pm j\ 0.095565)$$

Reconstruction Error: 47.4% for u

10.2% for w

8.9% for $\theta$

TABLE 6.3

Results of Longitudinal Dynamic Identification of a Six Man Submersive by MGRAM - u,w, and $\theta$ responses were generated by program DYNAMIC using Linear Coefficients only.

Identification Order N = 6

Depth Change Maneuver from 200 to 300 feet.

---

Identified transfer functions

$$h(s) = \frac{1}{D(s)} \begin{bmatrix} -0.000371s^5 & -0.000234s^4 & -0.000285s^3 -0.00000799s & -0.0000208s^2 -0.000000152 \\ -0.0297s^5 & -0.00790s^4 & -0.00449s^3 -0.0000403s & -0.000735s^2 -0.000000487 \\ -0.281s^5 & -0.212s^4 & -0.0808s^3 -0.00405s & -0.0282s^2 -0.000151 \end{bmatrix}$$

$$D(s) = s^6 + 0.25405s^5 + 0.15750s^4 + 0.025891s^3 + 0.0021218s^2 + 0.00032753s + 0.000014630$$

$$= (s + 0.040930 \pm j\, 0.35744)(s - 0.018657 \pm j\, 0.11361)(s + 0.055291)(s + 0.15422)$$

| | |
|---|---|
| Reconstruction Error: | 4.0% for u |
| | 5.4% for w |
| | 0.91% for $\theta$ |

## TABLE 6.4

Results of Longitudinal Dynamic Identification of a Six Man Submersive by MGRAM - u, w, and θ responses were generated by program DYNAMIC using <u>Linear and Nonlinear Coefficients</u>.

Identification Order N = 6

Depth Change Maneuver from 200 to 300 feet.

---

Identified transfer functions

$$h(s) = \frac{1}{D(s)} \begin{bmatrix} -0.000875s^5 & -0.000514s^4 & -0.000556s^3 & -0.0000303s^2 \\ & & -0.0000112s & -0.000000223 \\ -0.0313s^5 & -0.00998s^4 & -0.00350s^3 & -0.000682s^2 \\ & & -0.0000318s & -0.000000341 \\ -0.281s^5 & -0.212s^4 & -0.0808s^3 & -0.0282s^2 \\ & & -0.00405s & -0.000151 \end{bmatrix}$$

$$D(s) = s^6 + 0.29811s^5 + 0.11050s^4 + 0.023124s^3 + 0.00099258s^2 + 0.00019881s + 0.0000064645$$

$$= (s + 0.031807 \pm j\ 0.29261)(s - 0.015755 \pm j\ 0.095482)(s + 0.034405)(s + 0.23160)$$

| | | |
|---|---|---|
| Reconstruction Error: | 2.5% | for u |
| | 4.9% | for w |
| | 1.6% | for θ |

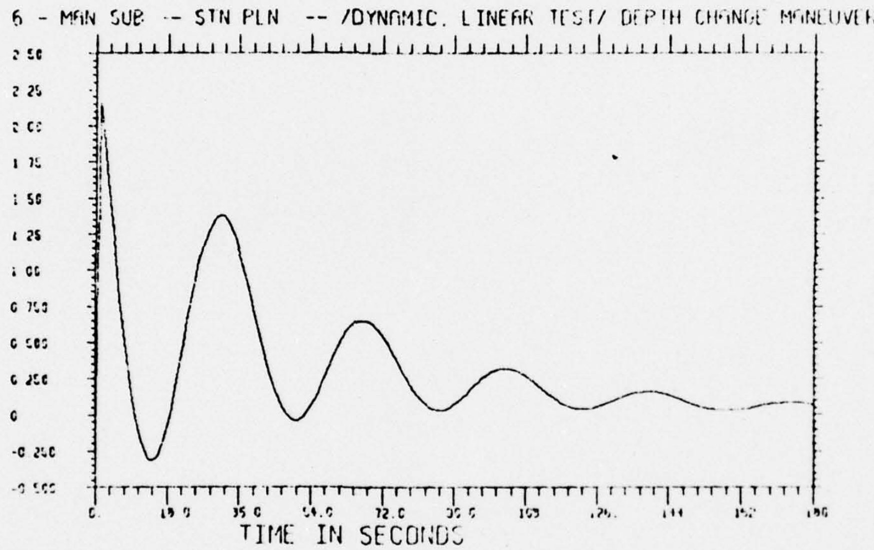



Fig. 30. Stern plane deflection. Vehicle simulated via coefficient set A.

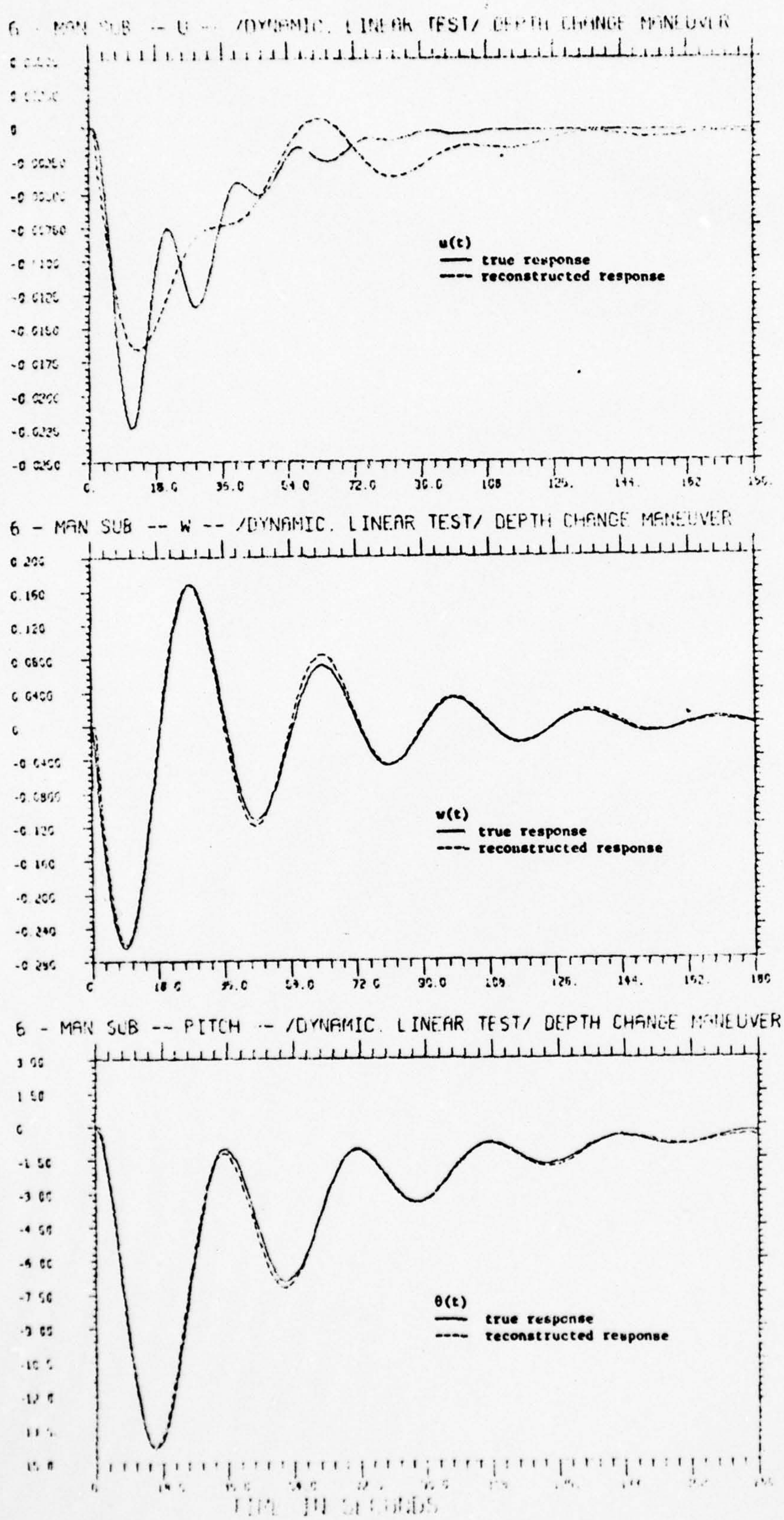



Fig. 31. Longitudinal response vs. response reconstructed from 4th order identification. Vehicle simulated via coefficient set A.

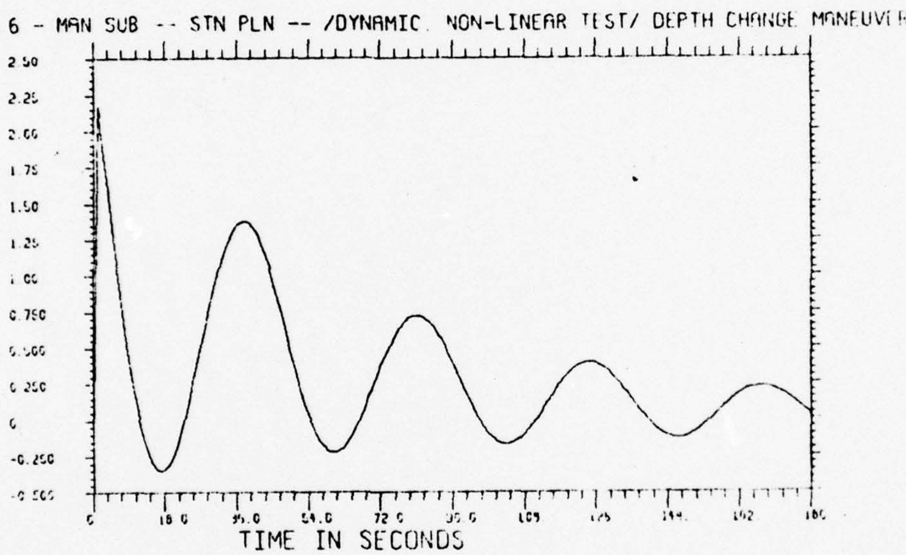



**Fig. 32. Stern plane deflection. Vehicle simulated via coefficient set B.**

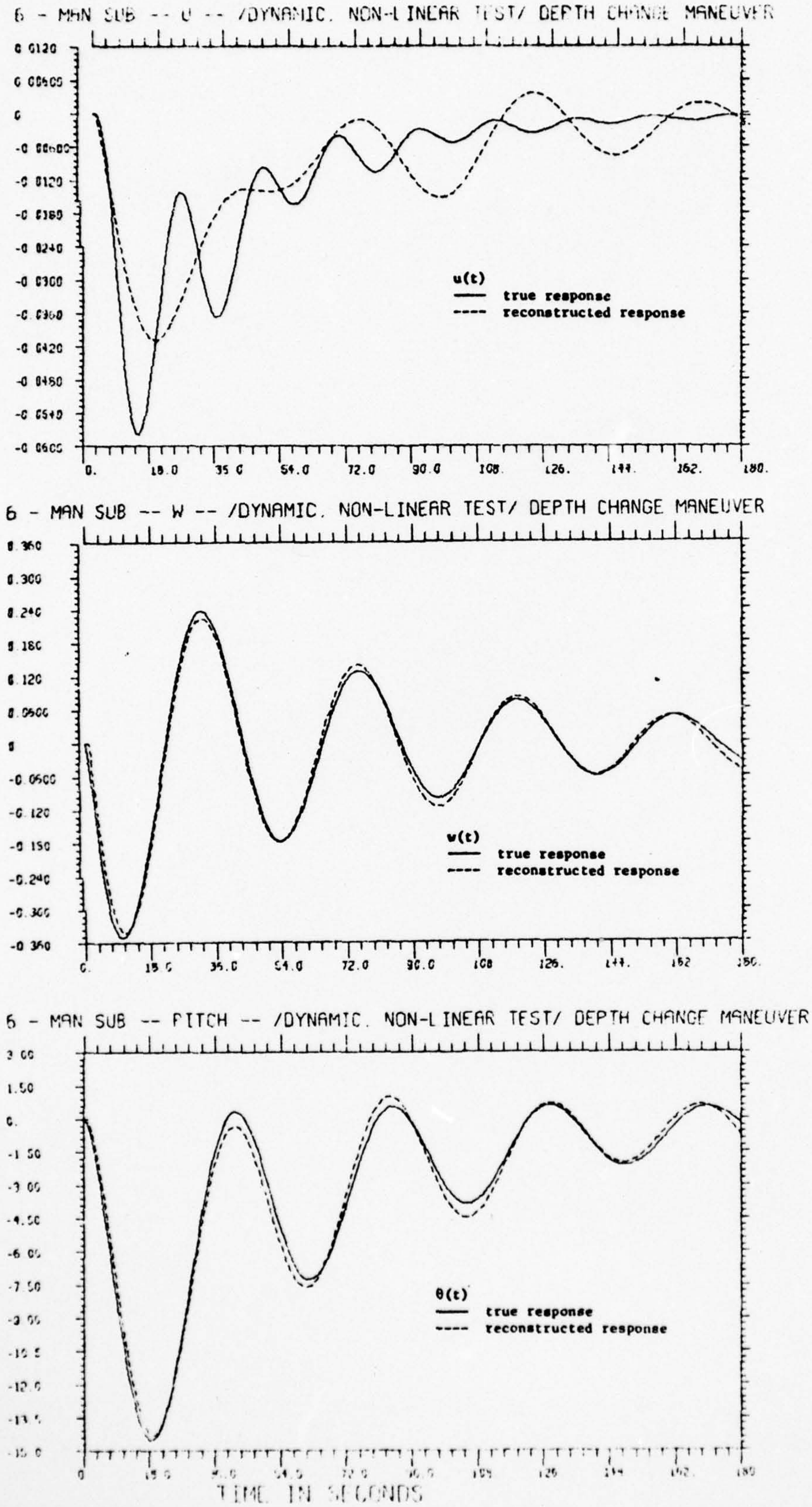



Fig. 33. Longitudinal responses vs. responses reconstructed from 4th order identification. Vehicle simulated via coefficient set B.

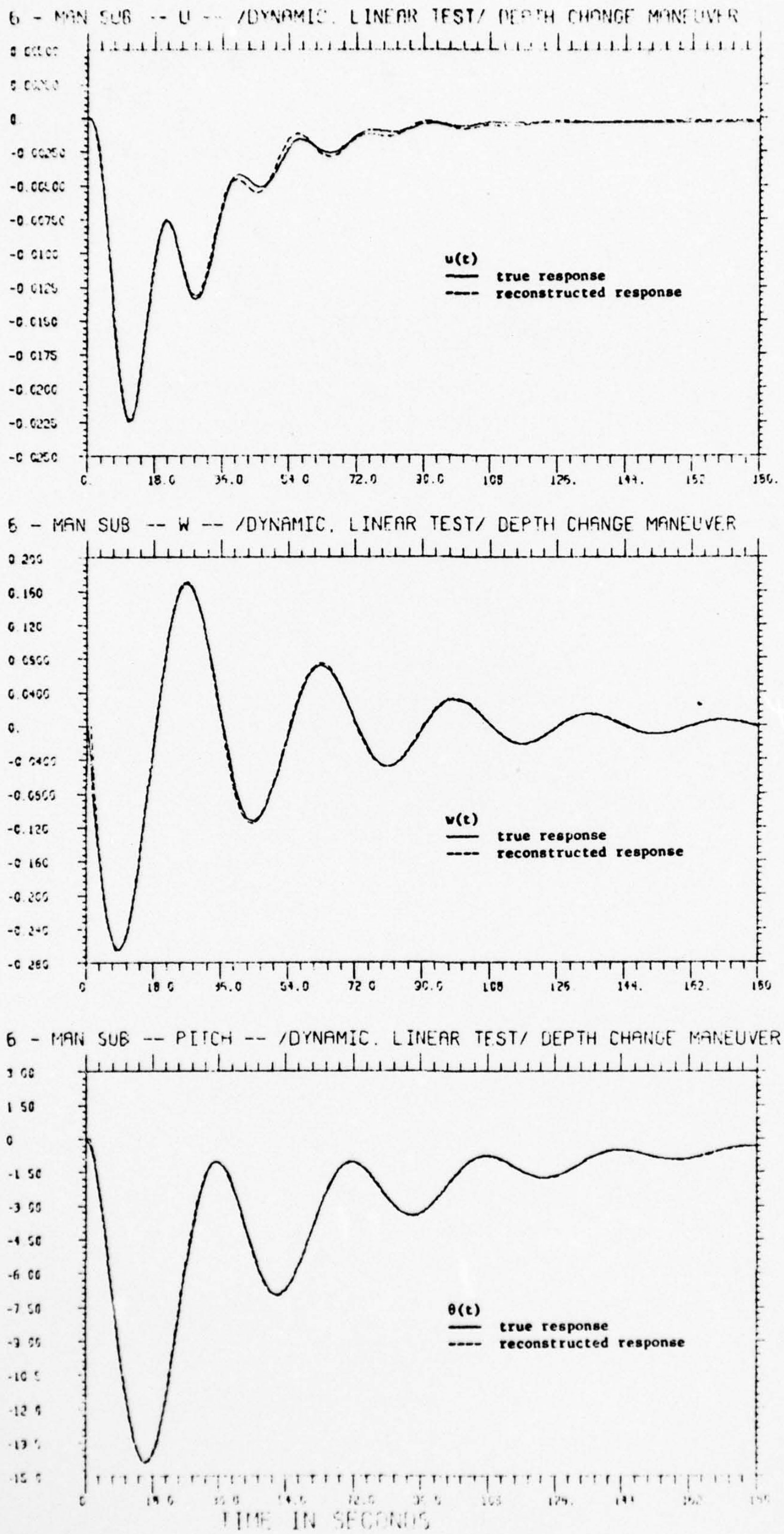



Fig. 34. Longitudinal response vs response reconstructed from 6th order identification. Vehicle simulated via coefficient set A.

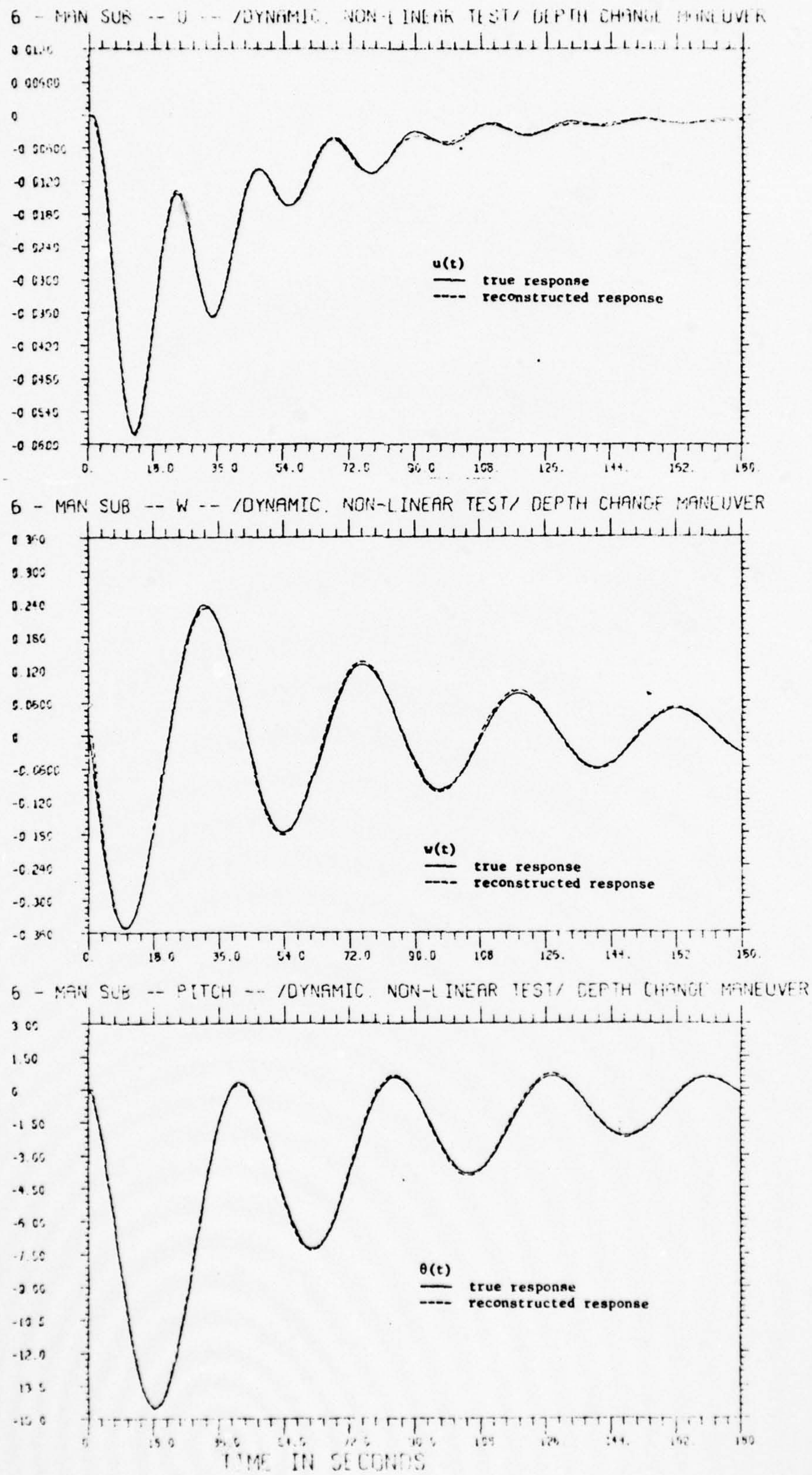



Fig. 35. Longitudinal response vs response reconstructed from 6th order identification. Vehicle simulated via coefficient set B.

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## APPENDIX A

The NASA computer program written by L. W. Taylor and K. W. Iliff [3] uses a modified Newton Raphson method for system identification. This program has been adapted for implementation on the IBM 360/65 computer of the University of South Florida. Because of some apparent discrepencies between the thoery and the program copy supplied to us by the Naval Coastal Systems Laboratory, we have incorporated some modifications. The computer program, now operational, is used for the GRAM-TAYLOR interface discussed in Chapter V.

To explain the modifications done we consider the computation of the gradient of the output with respect to the vector of unknown parameters. The notation used is largely the same as in [3]. Let:

$$\dot{x} = Ax + Bu \qquad \text{state equation} \qquad (1)$$

$$y = \begin{bmatrix} y^1 \\ y^2 \end{bmatrix} = \begin{bmatrix} x + b \\ Fx + Gu + d \end{bmatrix} \qquad \text{output} \qquad (2)$$

$$x(0) = \bar{x}(0) + q \qquad \text{initial state} \qquad (3)$$

$\bar{x}(0)$ given

Let c denote the vector of all unknown parameters. To use the recursion formula to improve an estimate $c_n$ of c, we need the gradient $\nabla_c y$. For this purpose let us differentiate (1) with respect to c:

$$\frac{\partial \dot{x}}{\partial c} = A \frac{\partial x}{\partial c} + \frac{\partial A}{\partial c} x + \frac{\partial B}{\partial c} u \qquad (4)$$

Letting the subscript k denote the sampled values of the variables (for example $y_i = y(k\Delta)$, where $\Delta$ is the sampling interval), and assuming a stair case approximation for the continuous time variables we have the following solution for equation (4):

$$\left(\frac{\partial x}{\partial c}\right)_{k+1} = \Phi\left(\frac{\partial x}{\partial c}\right)_k + \Psi\left[\frac{\partial A}{\partial c} x_k + \frac{\partial B}{\partial c} u_k\right] \tag{5}$$

where

$$\Phi = e^{A\Delta}, \quad \Psi = \int_0^{\Delta} e^{At}\, dt \tag{6}$$

On the other hand, by direct differentiation of (2) we have

$$\left(\frac{\partial y^1}{\partial c}\right)_{k+1} = \left(\frac{\partial x}{\partial c}\right)_{k+1} + \frac{\partial b}{\partial c} \tag{7a}$$

$$\left(\frac{\partial y^2}{\partial c}\right)_{k+1} = F\left(\frac{\partial x}{\partial c}\right)_{k+1} + \frac{\partial F}{\partial c} x_{k+1} + \frac{\partial G}{\partial c} u_{k+1} + \frac{\partial d}{\partial c} \tag{7b}$$

<u>Simplifications</u> - We now recognize that the vector c has the form

$$c = (p, q, b, d)$$

where p denotes the subset denoting the unknown entries from the matrices A, B, F and G. The vectors q, b, d were defined earlier in equations (1) to (3). Since

$$\frac{\partial y}{\partial c} = \begin{bmatrix} \frac{\partial y^1}{\partial p} & \frac{\partial y^2}{\partial p} \\ \frac{\partial y^1}{\partial q} & \frac{\partial y^2}{\partial q} \\ \frac{\partial y^1}{\partial b} & \frac{\partial y^2}{\partial b} \\ \frac{\partial y^1}{\partial d} & \frac{\partial y^2}{\partial d} \end{bmatrix} \tag{8}$$

we shall compute each entry on the right hand side of (8) in as simple a form as possible.

<u>First Row of (8)</u> - From (5) and (7),

$$\left(\frac{\partial x}{\partial p}\right)_{k+1} = \Phi\left(\frac{\partial x}{\partial p}\right)_k + \Psi\left[\frac{\partial A}{\partial p} x_k + \frac{\partial B}{\partial p} u_k\right], \tag{9a}$$

$$\left(\frac{\partial x}{\partial p}\right)_0 = \frac{\partial x(0)}{\partial p} = 0 \tag{9b}$$

$$(\frac{\partial y^1}{\partial p})_{k+1} = (\frac{\partial x}{\partial p})_{k+1} = \Phi (\frac{\partial y^1}{\partial p})_k + \Psi \quad [\frac{\partial A}{\partial p} x_k + \frac{\partial B}{\partial p} u_k], \tag{9c}$$

$$(\frac{\partial y^1}{\partial p})_0 = 0 \tag{9d}$$

$$(\frac{\partial y^2}{\partial p})_{k+1} = F(\frac{\partial x}{\partial p})_{k+1} + \frac{\partial F}{\partial p} x_{k+1} + \frac{\partial G}{\partial p} u_{k+1}$$

$$= F(\frac{\partial y^1}{\partial p})_{k+1} + \frac{\partial F}{\partial p} x_{k+1} + \frac{\partial G}{\partial p} u_{k+1} \tag{9e}$$

Initial value for this can be determined using (9d) and other known quantities.

Second Row of (8) - From (5) and (7),

$$(\frac{\partial x}{\partial q})_{k+1} = \Phi (\frac{\partial x}{\partial q})_k, \qquad (\frac{\partial x}{\partial q})_0 = I \text{ identity matrix} \tag{10a}$$

$$(\frac{\partial y^1}{\partial q})_{k+1} = \Phi (\frac{\partial y^1}{\partial q})_k, \qquad (\frac{\partial y^1}{\partial q})_0 = I \tag{10b}$$

$$(\frac{\partial y^2}{\partial q})_{k+1} = F (\frac{\partial y^1}{\partial q})_{k+1} \tag{10c}$$

Third Row of (8) - From (5) and (7),

$$(\frac{\partial x}{\partial b})_{k+1} = \Phi (\frac{\partial x}{\partial b})_k, \qquad (\frac{\partial x}{\partial b})_0 = 0 \tag{11a}$$

$$(\frac{\partial y^1}{\partial b})_{k+1} \equiv I \tag{11b}$$

$$(\frac{\partial y^2}{\partial b})_{k+1} \equiv 0 \tag{11c}$$

Fourth Row of (8) - From (5) and (7),

$$(\frac{\partial x}{\partial d})_{k+1} = \Phi (\frac{\partial x}{\partial d})_k, \qquad (\frac{\partial x}{\partial d})_0 = 0 \tag{12a}$$

$$(\frac{\partial y^1}{\partial d})_{k+1} \equiv 0 \tag{12b}$$

$$(\frac{\partial y^2}{\partial d})_{k+1} \equiv I \tag{12c}$$

Discussion - Equation (9) through (12) should have been used to compute the gradient $(\frac{\partial y}{\partial c})_{k+1}$ recursively. For the most part, the copy of the NASA program supplied to us did accomplish this recursion correctly. However, the following errors were found and corrected by us. They are discussed here for the benefit of the many other users of the Taylor program

| Earlier | Now |
|---|---|
| 1. The program computed $y^1_{k+1} = x_{k+1}$, $y^2_{k+1} = Fx_{k+1} + Gu_{k+1}$ | It now computes $y^1_{k+1} = x_{k+1} + b$, $y^2_{k+1} = Fx_{k+1} + Gu_{k+1} + d$ |
| 2. To compute $(\frac{\partial y^2}{\partial p})_{k+1}$, the earlier program was using $(\frac{\partial y^1}{\partial p})_k$ in equation (9e). | It now substitutes $(\frac{\partial y^1}{\partial p})_{k+1}$ into the right hand side of (9e). |
| 3. To compute $(\frac{\partial y^2}{\partial q})_{k+1}$, the program used $(\frac{\partial y^1}{\partial q})_k$ in equation (10c). | It now substitutes $(\frac{\partial y^1}{\partial q})_{k+1}$ into the right hand side of (10c) |
| 4. $x(0) = \bar{x}\ (0) + q \quad - b$ | This is changed to $x(0) = \bar{x}\ (0) + q$ |
| 5. updating of $\frac{\partial y^1}{\partial q}$ was in error | This is corrected to conform to (10b) |
| 6. Updating of parameters was modified: $c_{i+1} = c_i - \left[\nabla^2_c J_i\right]^{-1} \left[\nabla_c J_i\right]$ and the error function, J, is given by $J_i = \sum_{k=1}^{K} \lVert z_k - y_k \rVert^2_{D1}\Big\vert_i$ | Modification: $c_{i+1} = c_i - \alpha \left[\nabla^2_c J_i\right]^{-1} \left[\nabla_c J_i\right]$ where *at each iteration* $\alpha$ is choosen to be the largest number in the set, {1, 1/2, ..., 1/1024}, which decreases the error function from the previous iteration. If an $\alpha$ cannot be found in the above set, then convergence is assummed. Thus at each iteration $J_{i+1} \leq J_i$. |

## APPENDIX B

## AN APPROACH TO UNKNOWN NOISE IN MEASUREMENTS

In Chapter II, vehicle dynamics identification was carried out with noisy measurements. However, the characteristics of the noise, or error, process were assumed to be known. The purpose of this section is to propose an approach for the case when information about the noise process (the vector e(k) in (1c), or e(z) in (11)) is lacking. Specifically, a method is proposed to iteratively estimate the correlation matrix P of equation (25). There is as yet no theoretical basis for the procedure, nor does it always satisfactorily converge to a P matrix. Indeed, for an ill-posed system identification problem, the procedure results in oscillatory behavior. Dramatic improvements have, however, resulted in cases where the identification problem is well posed (i.e. each mode of the system appears reasonably strongly in at least one of the observed outputs).

To start the method, the white noise assumption, as discussed in subsection 2.3.2 of Chapter II, is used to estimate the initial (zeroth iterate) parameter vector $\gamma^{[0]}$. This vector is next used to determine the system response, and in turn the error

$$e^{[1]}(k) = y(k) - y^{[0]}(k)$$

This error sequence is used to compute $P^{[1]}$. The formulas necessary for the purpose are (18). which define the matrix $E^{[k]}$, and (27). This estimate of error correlation matrix is used to determine a new estimate $\gamma^{[1]}$. Specifically, it is chosen as the eigenvector, with its first entry normalized to 1, corresponding to the minimum eigenvalue solution of

$$\gamma^{[1]T} G\gamma^{[1]} = \gamma^{[1]T} P^{[1]} \gamma^{[1]}$$

This solution can be used to generate a new error sequence $e^{[2]}$ and the procedure can be iterated until satisfactory convergence occurs.